Driven dissipative Majorana dark states and dark spaces

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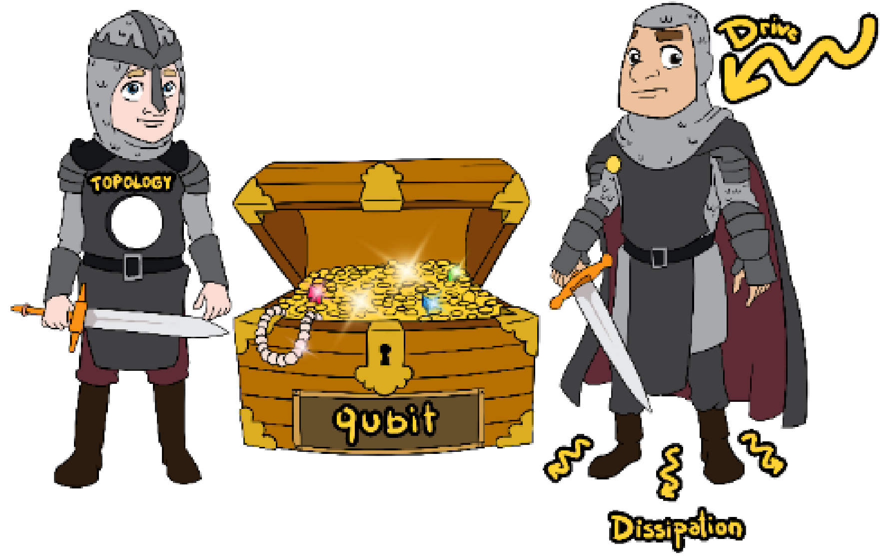
Overview



- **Motivation**
- **Driven Dissipative (DD) Majorana box qubits (MBQs): stabilization of dark states in a topologically protected system**

  Gau, RE, Zazunov & Gefen, PRB ***102****, 134501 (2020)*
- **Towards dark spaces: Dark Majorana Qubit**

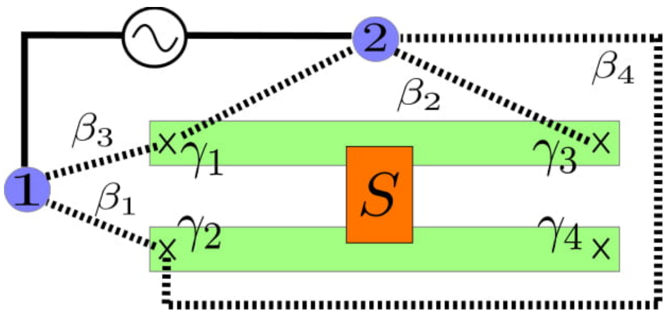
  Gau, RE, Zazunov & Gefen, PRL ***125****, 147701 (2020)*
- **Perspectives for future work**

Motivation: DD + Topological Protection

- Quantum Error Correction (QEC)
  - Autonomous („passive“) QEC for topological qubits
  - Topological codes (e.g. Majorana surface code) show exceptional fault tolerance → avoid overhead of active QEC ?
  - Theoretical basis vs realistic implementations?
  - …
- Enhanced robustness of dark spaces
  - Dark space = manifold of degenerate dark states
  - Residual noise (beyond the designed environment) detrimental since it may couple states within dark space
  - Natural suppression of unwanted noise by topological protection mechanisms
  - …

DD version of Majorana box qubit

- Dark state preparation & manipulation:
  - Two quantum dots (QDs) connected by AC driven tunnel link, with tunable tunnel couplings to a MBQ
  - Environmental electromagnetic phase fluctuations from circuit provide dissipation



- MBQ state dynamics
  - obeys **Lindbladian master equation**
  - simple strategies for optimization of purity, fidelity, and/or dissipative gap

Designed jump operators via steady-state unidirectional pump-cotunneling cycles

$\rightarrow$ exploit non-locality of Pauli op's in MBQ

$\rightarrow$ conspiracy of DD & topology leads to exceptional robustness

Majorana box

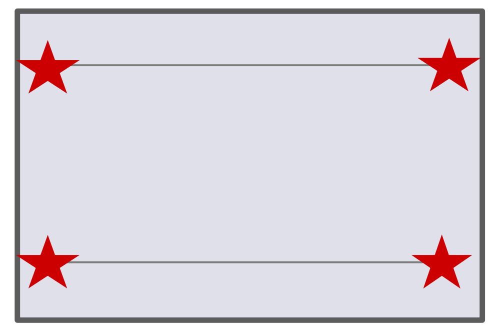
Béri & Cooper, PRL 2012
Altland & Egger, PRL 2013
Béri, PRL 2013
Altland, Béri, Egger & Tsvelik, PRL 2014

Two helical nanowires proximitized by same mesoscopic floating superconducting island

→ Energy scales below charging energy & proximity gap:

$$H_{\mathrm{box}} = E_C(\hat{N} - N_g)^2$$

gate parameter



- four Majorana zero modes (long wires)
  → two fermionic zero modes
- Condensate → bosonic zero mode
- Charging effects also protect against quasiparticle poisoning

Majorana box qubit (MBQ)

in Coulomb valley & for weak coupling to QDs

charge quantization → parity constraint

$$\gamma_1\gamma_2\gamma_3\gamma_4 = \pm 1$$

→ two-fold degenerate box ground state

(charge degrees of freedom are gapped out)

effective spin-1/2 „quantum impurity“ (qubit)

nonlocally encoded by topologically protected MBSs

→ long coherence times expected

Pauli operators: $X = i\gamma_1\gamma_3, \quad Y = i\gamma_3\gamma_2, \quad Z = i\gamma_1\gamma_2$

spin fractionalization

Plugge, Rasmussen, Egger & Flensberg, NJP 2017

QDs and driving field

Assume two QDs weakly coupled to MBQ

At low energy scales: one spinless fermion level for each QD

$$H_{\mathrm{d}} = \sum_{j=1,2} \epsilon_j d_j^\dagger d_j \qquad \epsilon_2 > \epsilon_1$$

QDs connected by **driven tunnel link** (AC gate voltage)

$$H_{\mathrm{drive}}(t) = w(t) d_1^\dagger d_2 + \mathrm{H.c.}, \qquad w(t) = t_{12} + 2A\cos(\omega_0 t)$$

small static couplings

$\rightarrow$ no qualitative changes

$\rightarrow$ here study $t_{12} = 0$

drive amplitude

drive frequency

Single-occupancy regime

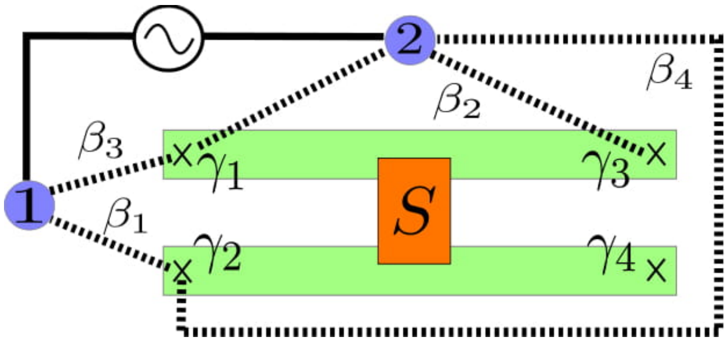
- On timescales $\delta t > 1/E_C$: number of electrons on Majorana box conserved $\rightarrow$ total occupany of QDs also conserved

  $\rightarrow$ assume: QDs initially loaded with one electron

  $\rightarrow$ Low-energy QD dynamics described by spin-1/2
- Pauli operators for QD sector:

$$\tau_+ = \tau_-^\dagger = d_1^\dagger d_2, \quad \tau_z = d_1^\dagger d_1 - d_2^\dagger d_2$$

Dissipation



Electromagnetic fluctuations

- Predominant effect: phase noise
- Appears via fluctuating phases in the tunneling amplitudes: P(E) theory

$$\hat{\lambda}_{j\nu} = \lambda_{j\nu} e^{i\theta_{j\nu}}$$

$$H_{\text{tun}} = t_0 e^{-i\hat{\phi}} \sum_{j,\nu} \hat{\lambda}_{j\nu} d_j^\dagger \gamma_\nu + \text{H.c.}$$

overall hybridization scale

Box phase operator

$$\max\{|\lambda_{j\nu}|\} = 1$$

- Electromagnetic environment due to circuitry → harmonic oscillator bath

$$H_{\text{env}} = \sum_m E_m b_m^\dagger b_m$$

$$\theta_{j\nu} = \sum_m g_{j\nu,m}(b_m + b_m^\dagger)$$

Bath spectral density

- Long wavelengths: only a single fluctuating phase appears!
- Ohmic spectral density $\mathcal{J}(\omega) = \alpha\omega e^{-\omega/\omega_c}$
- Phase correlations

$$\alpha = \frac{e^2}{2h}\mathrm{ReZ}(\omega = 0)$$

environmental impedance

$$J_{\mathrm{env}}(t) = \langle[\theta(t) - \theta(0)]\theta(0)\rangle_{\mathrm{env}}$$
$$= \int_0^\infty \frac{d\omega}{\pi}\frac{\mathcal{J}(\omega)}{\omega^2}$$
$$\times \left\{[\cos(\omega t) - 1]\coth\left(\frac{\omega}{2T}\right) - i\sin(\omega t)\right\}$$

integral can be done analytically…

Low-energy theory

- Parameter regime of interest:

$$\max\{T, A, t_0, \omega_0, \omega_c, |\epsilon_j|\} \ll \min\{E_C, \Delta\}$$

  - these parameters only affect speed of approach to dark space
  - State design parameters: complex-valued tunneling amplitudes λ
- Project to degenerate charge ground state sector

$$H_{\text{eff}}(t) = H_{\text{d}} + H_{\text{env}} + H_{\text{drive}}(t) + H_{\text{cot}}$$

$$H_{\text{cot}} = 2g_0(e^{i\theta}W_+\tau_+ + \text{H.c.}) + g_0 W_z \tau_z \qquad g_0 \equiv \frac{t_0^2}{E_C}$$

$$W_{jk} = \sum_{\mu<\nu}(\lambda_{j\nu}\lambda^*_{k\mu} - \lambda_{j\mu}\lambda^*_{k\nu})\gamma_\mu\gamma_\nu \qquad W_z = W_{11} - W_{22}, \quad W_+ \equiv W_{12}$$

→ linear combinations of MBQ Pauli op's (X,Y,Z)

Parameter regime of interest

- Switch to interaction picture w.r.t. $H_{\mathrm{d}} + H_{\mathrm{env}}$
- Assume **resonant driving** $\omega_0 = \epsilon_2 - \epsilon_1$

$$\tilde{H}_{\mathrm{drive}} = A(d_1^\dagger d_2 + d_2^\dagger d_1) = A\tau_x$$

- Parameter regime: $g_0 \ll T \ll \omega_0, \quad A \lesssim g_0$
  - First inequality → Born-Markov approximation
  - Second inequality → rotating wave approximation
  - For state stabilization & manipulation: weakly driven limit
- Density matrix (DM) of composite **QD+MBQ** system obeys **Lindblad master equation**

QD+MBS DM: Lindblad equation

$$\partial_t \rho(t) = -i[H_\mathrm{L}, \rho(t)] + \sum_{\pm} \Gamma_\pm \mathcal{L}[J_\pm]\rho(t)$$

$$\mathcal{L}[J]\rho = J\rho J^\dagger - \tfrac{1}{2}\{J^\dagger J, \rho\}$$

- **Eff. Hamiltonian** $H_\mathrm{L} = A\tau_x + g_0 W_z \tau_z + \sum_\pm h_\pm J_\pm^\dagger J_\pm$
- **Jump operators** $J_\pm = 2W_\pm \tau_\pm = J_\mp^\dagger$
- Dissipative transition rates $\Gamma_\pm = 2g_0^2 \,\mathrm{Re}\Lambda_\pm$
- Lamb shifts

  $h_\pm = g_0^2 \,\mathrm{Im}\Lambda_\pm$

$$\Lambda_\pm = \int_0^\infty dt\, e^{\pm i\omega_0 t} e^{J_\mathrm{env}(t)}$$

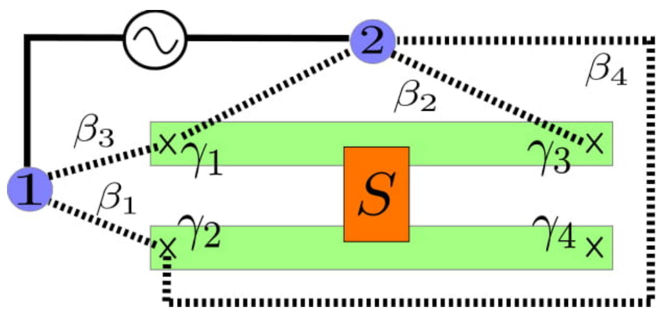
Unidirectional inelastic cotunneling

Analytic properties of bath correlation function $\rightarrow$ detailed balance

$$\Gamma_- = e^{-\omega_0/T}\,\Gamma_+, \qquad h_- = e^{-\omega_0/T}\,h_+$$

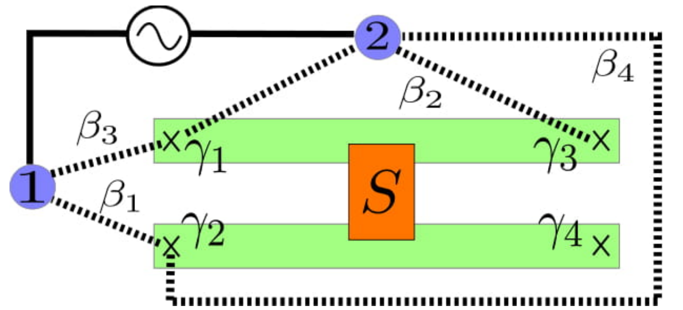
$\rightarrow$ Lindbladian dominated by a single jump operator!

$$J_+ = \tilde{J}_+ \tau_+$$



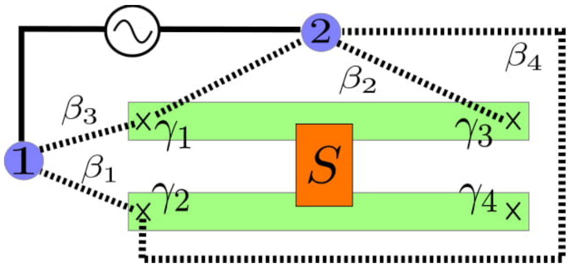
$$\tilde{J}_+ = 2ie^{i\beta_2}|\lambda_{23}|(e^{-i\beta_3}|\lambda_{11}|X - e^{-i\beta_1}|\lambda_{12}|Y) - 2i[e^{-i\beta_1}|\lambda_{12}\lambda_{21}| - e^{i\beta_4}|\lambda_{11}\lambda_{22}|]Z,$$

Intuitive picture



- Drive field pumps electron from QD1 to QD2
- Inelastic cotunneling only from QD2 back to QD1 via box (with action on MBQ state)
  - photon emission
  - backward step exponentially suppressed
- Weak driving: **unidirectional steady-state cycle** $\rightarrow$ autonomous MBQ state stabilization
- Design jump operator (and thus MBQ target state) by tuning QD-MBQ tunnel links

Example

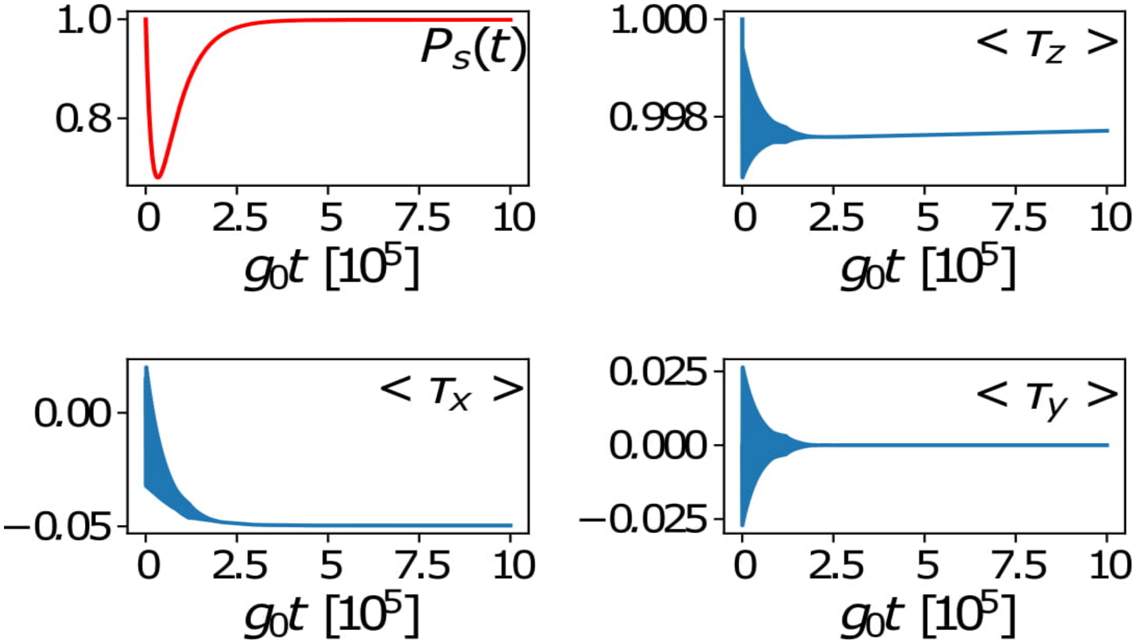


$$|\lambda_{11}| = |\lambda_{12}|, \quad \lambda_{22} = 0,$$

$$\beta_1 = -\beta_2 = \pi/2, \quad \beta_3 = \beta_4 = 0$$

$$\rightarrow J_+ = 2|\lambda_{11}|(2|\lambda_{23}|\sigma_+ + |\lambda_{21}|Z)\tau_+$$

- QD+MBS DM approaches ideal purity $P_s(t) = \mathrm{tr}\,\rho^2(t)$
- steady-state DM is independent of initial state

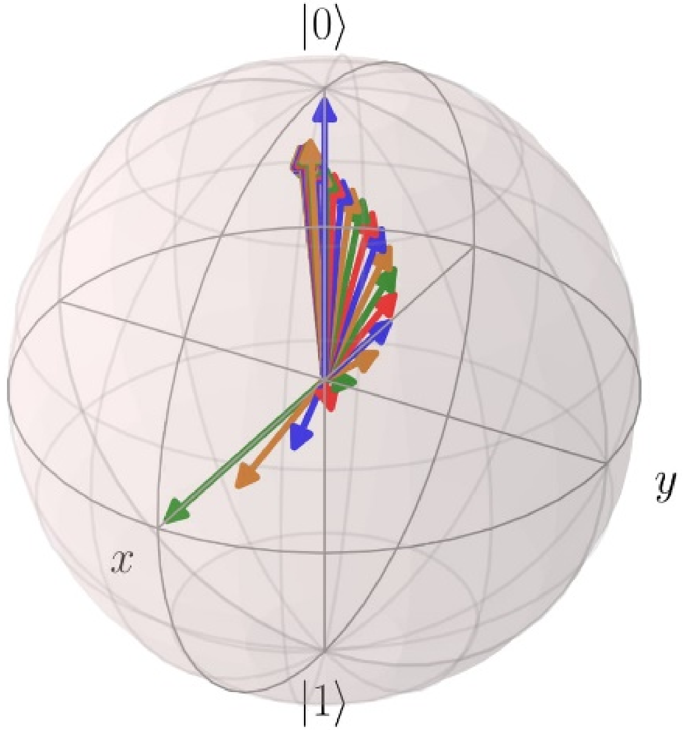


$T/g_0 = 4, \ \omega_0/g_0 = 40, \ \omega_c/g_0 = 200, \ A/g_0 = 0.1, \ \alpha = 1/4$ $\quad |\lambda_{11}| = |\lambda_{12}| = |\lambda_{23}| = 1$, and $|\lambda_{21}| = 0.1$

MBQ state dynamics

- Here: starting from X=+1 eigenstate, MBQ is steered towards pure state |0>
- Finite weight in QD2 $p \approx A/\omega_0$
- Numerics: QD+MBS DM not entangled at long times!

$$\rho(t \to \infty) \simeq \rho_{\mathrm{M}}(t) \otimes \rho_{\mathrm{d}}(t)$$



After initial transients: results also follow from Lindblad equation for MBQ sector only!

Jump operators: $\tilde{J}_{\pm}$

Dissipative rates: $\tilde{\Gamma}_{+} = p\Gamma_{+}, \quad \tilde{\Gamma}_{-} = (1-p)\Gamma_{-}$

Dark state stabilization

State design parameters for reaching desired target state
↔ vanishing eigenvalue of Lindbladian

Speed of approach ↔ dissipative gap (= real part of smallest non-zero eigenvalue)

Example: Z=±1 eigenstates obtained for

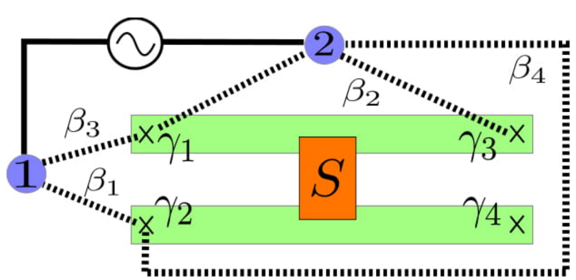
$$|\lambda_{11}| = |\lambda_{12}|, \quad \lambda_{21} = \lambda_{22} = 0, \quad \beta_1 - \beta_3 = \pm\pi/2$$

→ **Jump operator implements dissipative map** $\tilde{J}_+ \propto \hat{E}_\pm$

→ Dissipative dynamics maps every input state to Z=±1 eigenstate

$$\hat{E}_\pm = \sigma_\pm = (X \pm iY)/2$$

→ no obstruction to steering from effective Hamiltonian $\tilde{H}_L \propto Z$



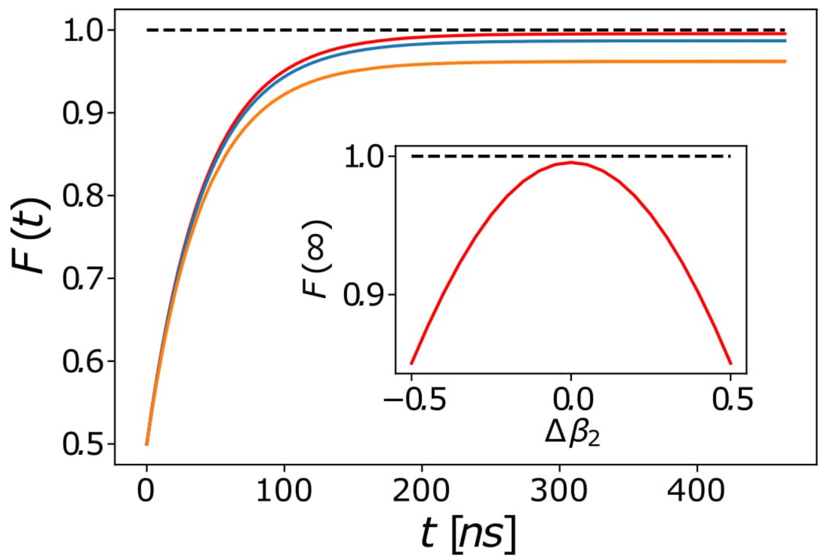
Robustness

Ex.: Magic state stabilization

$$|m\rangle = e^{-i\frac{\pi}{8}Y}|0\rangle$$

$$|\lambda_{12}| = |\lambda_{23}|, \quad |\lambda_{21}| = |\lambda_{11}| = |\lambda_{23}|/\sqrt{2},$$

$$\lambda_{22} = 0, \quad \beta_3 = \beta_1 + \beta_2, \quad \beta_2 = -\pi/2$$



$g_0/E_C = 2.5 \times 10^{-3}$

- Fidelity $F(t) = \mathrm{tr}[|\psi\rangle\langle\psi|\rho_M(t)]$
- Perfect fidelity for ideal parameters
- For 10% (or even 20%) deviations in all state design parameters: we still get F>0.9
  - Inset: fidelity vs mismatch in a single angle
  - For these parameters: Dissipative gap $\Delta_m^{-1} \simeq 80$ ns

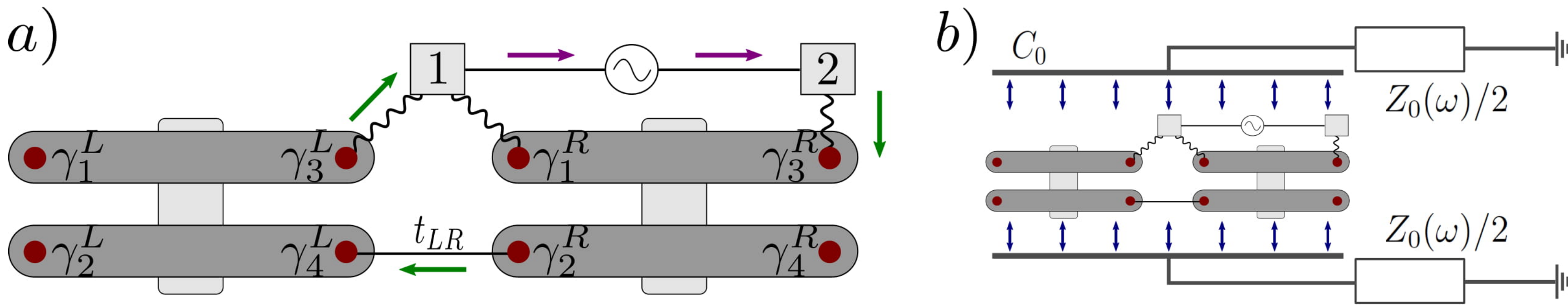
Dark state readout

- MBQ parity readout can be performed with same techniques as suggested before

  Plugge et al., NJP 2017, Karzig et al., PRB 2017

  - For instance, additional QD coupled to two Majorana states → capacitance spectroscopy
- Readout should decouple from stabilization!
  - typical measurement time scales are long against inverse dissipative gap
- Manipulation: vary state design parameters or employ single-electron pumping protocols

Towards Majorana Dark Spaces



$$X_\kappa = i\gamma_1^\kappa\gamma_3^\kappa, \qquad Y_\kappa = i\gamma_3^\kappa\gamma_2^\kappa, \qquad Z_\kappa = i\gamma_1^\kappa\gamma_2^\kappa$$

Here: DD-stabilized **„Dark Majorana Qubit“**

- two coupled MBQs with three QD-MBS tunnel links

DD mechanism adds extra protection layer

→ qubit states stabilized for indefinite times

→ perform quantum manipulations in dark space

Dark qubit space

Use $\beta_1 = \pi, \qquad |\lambda_{1,1R}| = \lambda_{LR}|\lambda_{1,3L}| \quad \rightarrow$ Dominant jump operator implements **dissipative map** $\hat{E}_{1,-}$

$$\hat{E}_{1,\pm} = (\mathbb{1} \pm Z_L Z_R) X_R$$

Bell states have $Z_L Z_R = \pm 1$ and $X_L X_R = \pm 1$

Even parity: $Z_L Z_R = +1$

$$|\psi_\pm\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}$$

$$\hat{E}_{1,-}|\psi_\pm\rangle = |\phi_\pm\rangle$$

Odd parity:

$$|\phi_\pm\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}$$

$$\hat{E}_{1,-}|\phi_\pm\rangle = 0$$

$\rightarrow$ **degenerate dark space $\leftrightarrow$ odd-parity Bell states**

No obstruction from Hamiltonian dynamics: $H_L \propto Z_L Z_R$

Dark Majorana qubit

- Pauli operators for our dark qubit can be chosen as

$$X_D = X_L X_R = -\gamma_1^L \gamma_3^L \gamma_1^R \gamma_3^R,$$

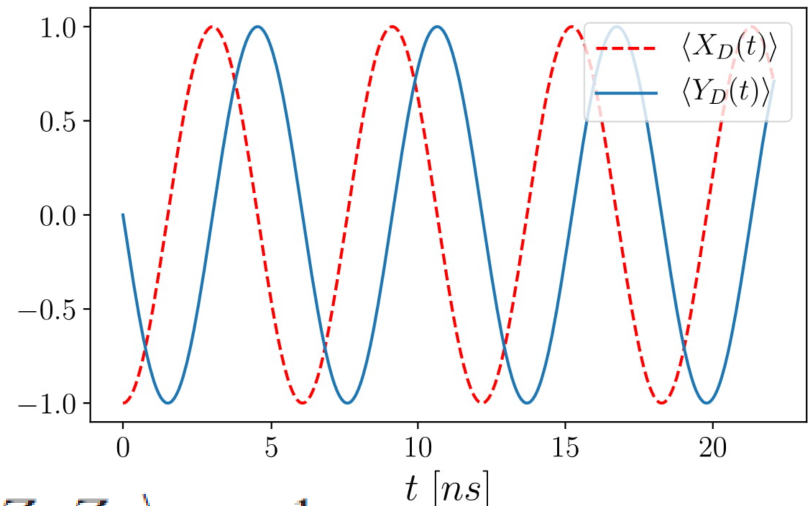
$$Y_D = Y_L X_R = \gamma_2^L \gamma_3^L \gamma_1^R \gamma_3^R, \qquad Z_D = Z_L = i\gamma_1^L \gamma_2^L$$

- DD induced fault tolerance: qubit states are stabilized for arbitrary times

State manipulation

- For evolving a dark state to some target state: break dark space degeneracy during intermediate times !
  - Challenge: avoid coupling to other Hilbert space sectors (e.g., even-parity Bell states) & preserve purity
  - Induce (slowly varying) tunnel coupling between two Majoranas, e.g.,

$$H_Z = iA_Z(t)\gamma_1^L\gamma_2^L = A_Z(t)Z_L$$



  - Dissipative dynamics with static perturbation:

    Rabi oscillations

Numerics (for all t): $\langle Z_D\rangle = 0 \quad \langle Z_L Z_R\rangle = -1$

Perspectives for future work

- Can such DD Majorana schemes be usefully scaled up to large (1D, 2D, 3D) MBQ networks ?
- Devise robust **Majorana braiding** protocols by working in a dark space manifold ?
- Quantum simulation applications, e.g., spin liquids, fracton phases ?
- theory of Majorana code with autonomous QEC → compare performance (fault tolerance, speed) to Majorana surface code using active QEC
- …

Steering MBQ states

with S. Morales, Y. Gefen, I. Gornyi, and A. Zazunov

Work in progress

Replace DD mechanism by sequence of steering protocols during time step δt

Roy, Chalker, Gornyi & Gefen, Phys. Rev. Res. (2020)

For preparing & stabilizing a desired target MBQ state

1. Initialize QD sector with one electron on a single QD
2. Switch on appropriately chosen (fixed) QD-MBQ tunnel couplings & let system evolve for one time step
3. Decouple QDs from MBQ → „blind measurement“ of final QD state vs „intelligent feedback“
4. Repeat steps 1 to 3 until convergence has been reached

Questions include:

Can one improve performance (speed of approach, robustness etc) by replacing blind measurements of final QD state by feedback, e.g. at certain rate ?

Kumar, Zirnstein, Snizhko, Gornyi & Gefen, arXiv:2101.07284

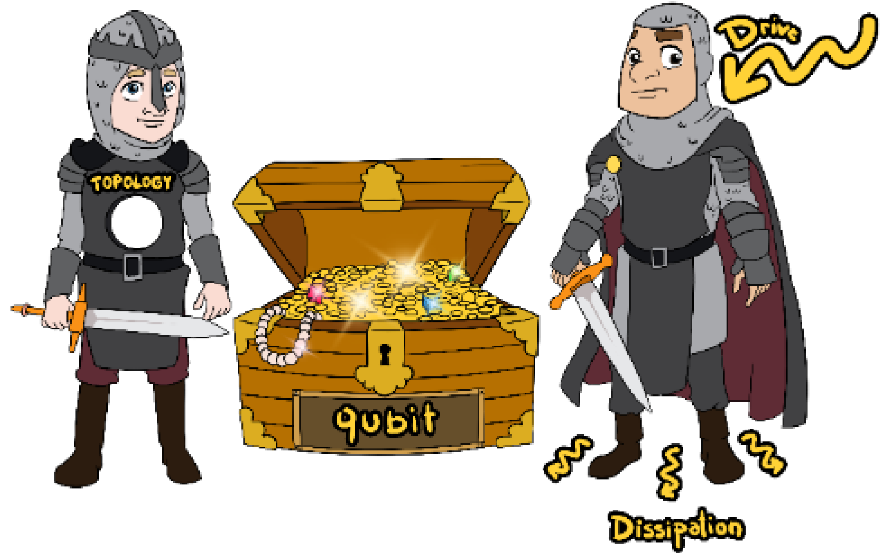
→ realize tunable crossover between active and passive QEC ?

→ properties of this crossover ?

My bullet points:

- Can DD schemes or combinations of passive and active QEC outperform active QEC (already for non-topological codes) ?
  - Scaling properties with system size?
  - Practical implemention of such codes?
  - Steering vs DD schemes?
- How powerful is the combination of DD and topological protection in general?
  - Beyond Majoranas: Parafermionic systems?

Conclusions



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- **Perspectives for future work**