# Driven dissipative Majorana dark states and dark spaces

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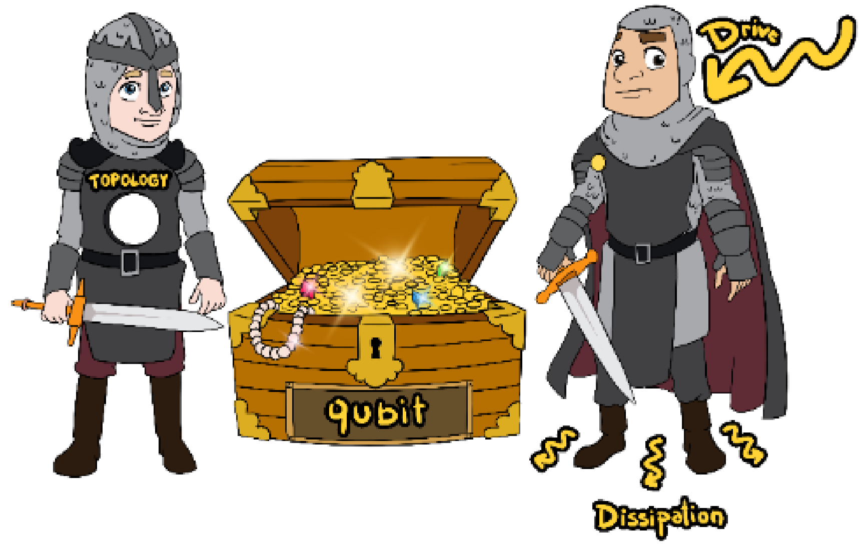
# Overview




- **Motivation**
- **Driven-Dissipative (DD) Majorana qubits: stabilization of dark states in a topologically protected system**

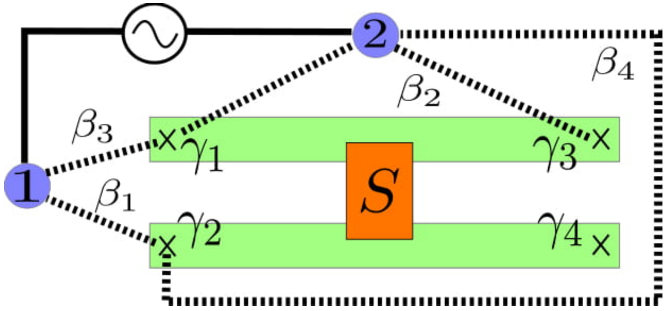
  *Gau, RE, Zazunov & Gefen, PRB **102**, 134501 (2020)*
- **Towards a dark space: Dark Majorana Qubit**

  *Gau, RE, Zazunov & Gefen, PRL **125**, 147701 (2020)*

# Motivation: DD + Topological Protection

- Quantum Error Correction (QEC)
  - Autonomous („passive“) QEC for topological qubits
  - Topological codes (e.g. Majorana surface code) show exceptional fault tolerance → avoid overhead of active QEC ?
  - Theoretical basis vs realistic implementations?
  - …
- Enhanced robustness of dark spaces
  - Residual noise (beyond the designed environment) detrimental since it may couple states within dark space
  - Natural suppression of unwanted noise by topological protection mechanisms
  - …

# DD version of Majorana box qubit




- Basic setup:
  - Two **quantum dots (QDs)** connected by AC driven tunnel link, with tunable tunnel couplings to a **Majorana qubit**
  - Environmental electromagnetic phase fluctuations from electric circuit provide dissipation
- In well defined parameter regime, Majorana state dynamics obeys **Lindbladian master equation**
  - **Dark state = decoherence free subspace with zero eigenvalue of the Lindbladian**
  - simple strategies for optimization of purity, fidelity, and speed of approach to dark state

# Model

Box: charge quantization → parity constraint $\gamma_1\gamma_2\gamma_3\gamma_4 = \pm 1$

→ two-fold degenerate ground state: **Majorana qubit**

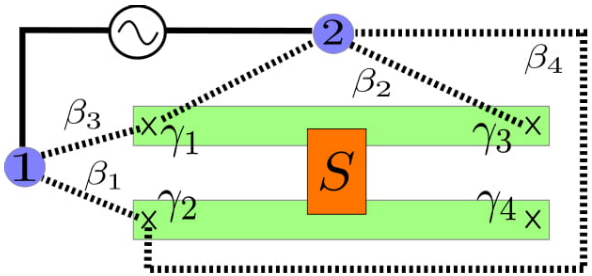
nonlocally encoded by topologically protected Majorana operators, Pauli operators

$$X = i\gamma_1\gamma_3, \quad Y = i\gamma_3\gamma_2, \quad Z = i\gamma_1\gamma_2$$

Quantum dots are operated in single-electron regime, resonant drive frequency $\omega_o$ and weak drive amplitude *A*

→ QD dynamics described by another set of Pauli operators

# Dissipation




## Electromagnetic fluctuations

- Appears via fluctuating phases in tunneling amplitudes: P(E) theory

$$\hat{\lambda}_{j\nu} = \lambda_{j\nu} e^{i\theta_{j\nu}}$$

- Electromagnetic environment due to electric circuit → Ohmic harmonic oscillator bath

$$\mathcal{J}(\omega) = \alpha \omega e^{-\omega/\omega_c}$$

$$\alpha = \frac{e^2}{2h} \mathrm{Re} Z(\omega = 0)$$

**environmental impedance**

# Parameter regime

Electron transfer between QDs through box proceeds by inelastic cotunneling

QD Pauli operators

$$H_{\text{cot}} = 2g_0(e^{i\theta} W_+ \tau_+ + \text{H.c.}) + g_0 W_z \tau_z$$

Environmental noise

linear combinations of (X,Y,Z)

Parameter regime of interest:

$$g_0 \ll T \ll \omega_0, \quad A \lesssim g_0$$

- First inequality → Born-Markov approximation
- Second inequality → unidirectionality & rotating wave approx
- For state stabilization & manipulation: **weakly driven** limit

# Lindblad equation

Density matrix of QDs & Majorana qubit then obeys **Lindblad master equation**

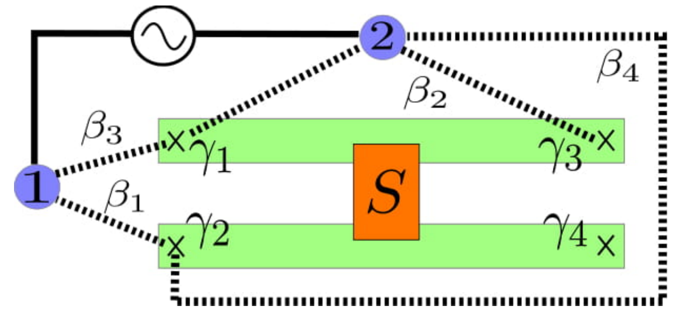
$$\partial_t \rho(t) = -i[H_{\mathrm{L}}, \rho(t)] + \sum_{\pm} \Gamma_{\pm} \mathcal{L}[J_{\pm}]\rho(t)$$

$$\mathcal{L}[J]\rho = J\rho J^{\dagger} - \tfrac{1}{2}\{J^{\dagger}J, \rho\}$$

At low T: cotunneling with photon emission dominates,

only a single jump operator matters $\Gamma_{-} = e^{-\omega_0/T}\Gamma_{+}$

# Intuitive picture




- Drive field pumps electron from QD1 to QD2
- Inelastic cotunneling only from QD2 back to QD1 via box, acting on Majorana state
  - photon emission
  - backward step exponentially suppressed
- Weak driving: **unidirectional steady-state cycle** $\rightarrow$ autonomous Majorana dark state stabilization
- One can design the jump operator (and thus the target state) via QD-Majorana tunneling amplitudes!

# Dark state stabilization

Tunneling parameters for desired target state ↔ vanishing eigenvalue of Lindbladian

Speed of approach ↔ dissipative gap (= real part of smallest non-zero eigenvalue)

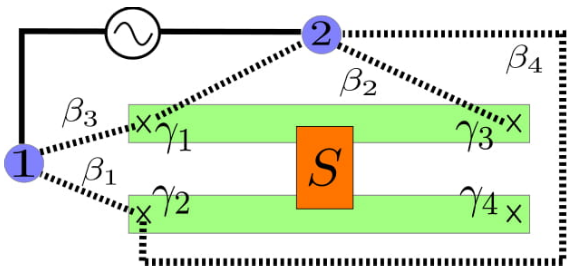
Ex: Z=±1 eigenstates follow for

$$|\lambda_{11}| = |\lambda_{12}|, \quad \lambda_{21} = \lambda_{22} = 0, \quad \beta_1 - \beta_3 = \pm\pi/2$$

→ **Jump operator = dissipative map** $\hat{E}_\pm = \sigma_\pm = (X \pm iY)/2$

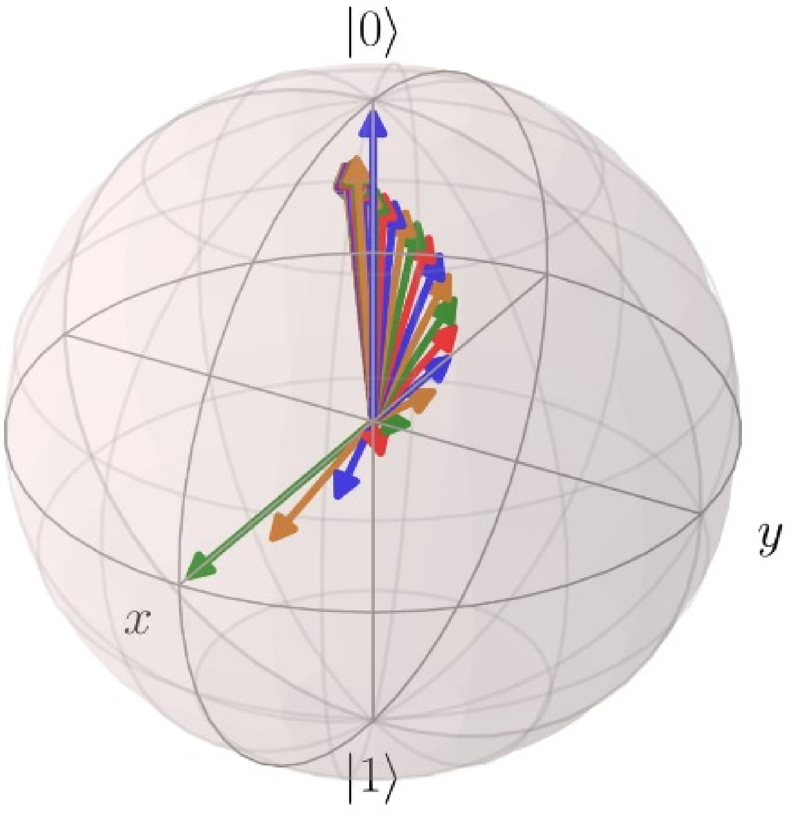
→ Dissipative dynamics maps every input state to Z=±1 state

→ no obstruction to steering from effective Hamiltonian $\tilde{H}_L \propto Z$



# Majorana dark state

- start from X=+1 initial state: DD steering to dark state |0>
  - Steady state is independent of initial state
- Stabilization persists as long as DD protocol is active: **fault tolerant quantum state memory**



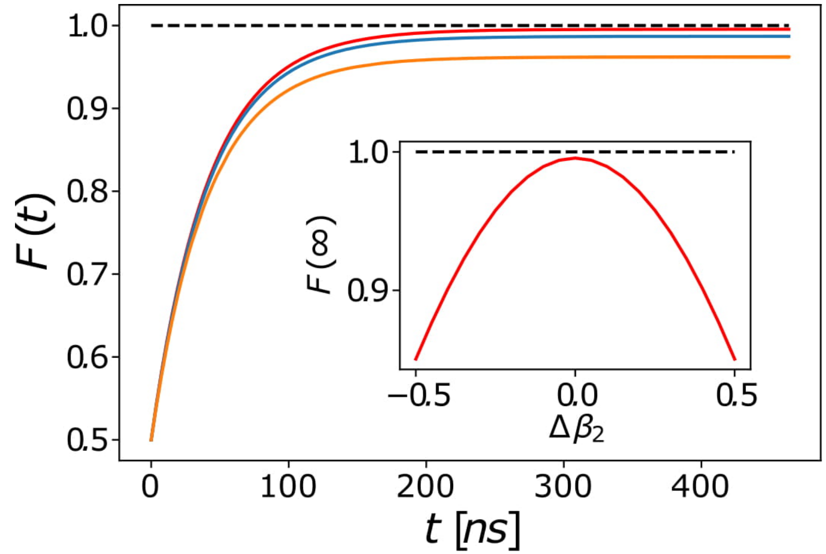

$$T/g_0 = 4,\ \omega_0/g_0 = 40,\ \omega_c/g_0 = 200,\ A/g_0 = 0.1,\ \alpha = 1/4$$

# Robustness

**Ex.: Magic state stabilization**

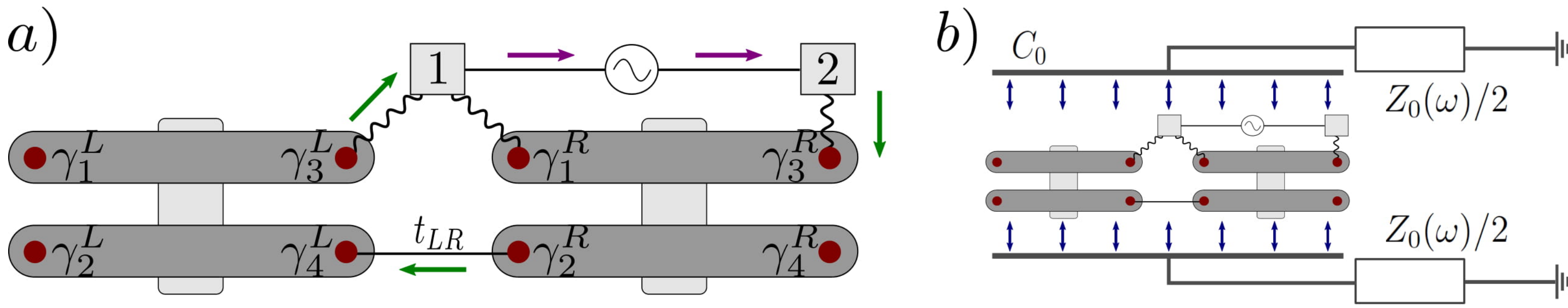
$$|m\rangle = e^{-i\frac{\pi}{8}Y}|0\rangle$$

$$|\lambda_{12}| = |\lambda_{23}|, \quad |\lambda_{21}| = |\lambda_{11}| = |\lambda_{23}|/\sqrt{2},$$
$$\lambda_{22} = 0, \quad \beta_3 = \beta_1 + \beta_2, \quad \beta_2 = -\pi/2$$




- Fidelity $\quad F(t) = \mathrm{tr}[|\psi\rangle\langle\psi|\rho_M(t)]$
- Perfect fidelity for ideal parameters
- For 10% (or even 20%) deviations in all parameters: we still get F>0.9
  - For these parameters: Dissipative gap $\Delta_m^{-1} \simeq 80$ ns

# Towards Majorana Dark Spaces




$$X_\kappa = i\gamma_1^\kappa\gamma_3^\kappa, \qquad Y_\kappa = i\gamma_3^\kappa\gamma_2^\kappa, \qquad Z_\kappa = i\gamma_1^\kappa\gamma_2^\kappa$$

Here: DD-stabilized **„Dark Majorana Qubit“**

- two coupled boxes with three QD-Majorana tunnel links

DD mechanism adds extra protection layer

→ qubit states stabilized for indefinite times

→ perform quantum manipulations in dark space!

# Dark Majorana qubit space

Dominant jump operator implements **dissipative map** $\hat{E}_{1,-}$

$$\hat{E}_{1,\pm} = (\mathbb{1} \pm Z_L Z_R) X_R$$

**Bell states** have $Z_L Z_R = \pm 1$ and $X_L X_R = \pm 1$

Even parity:

$$|\psi_\pm\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}$$

$$\hat{E}_{1,-}|\psi_\pm\rangle = |\phi_\pm\rangle$$

**Odd parity:**

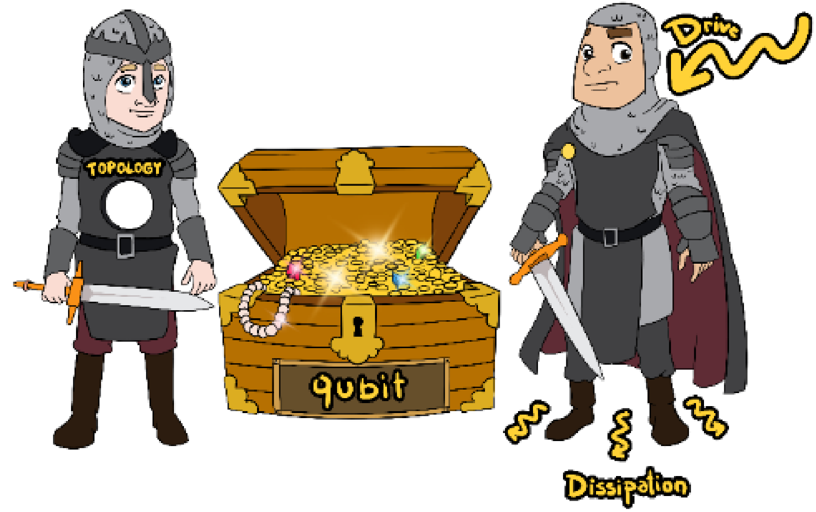
$$|\phi_\pm\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}$$

$$\hat{E}_{1,-}|\phi_\pm\rangle = 0$$

→ **degenerate dark space ↔ odd-parity Bell states**

No obstruction from Hamiltonian dynamics: $H_L \propto Z_L Z_R$

# Conclusions

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