# Non-Abelian anyon statistics through AC conductance of a Majorana interferometer

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CRC 183 ENTANGLED STATES OF MATTER

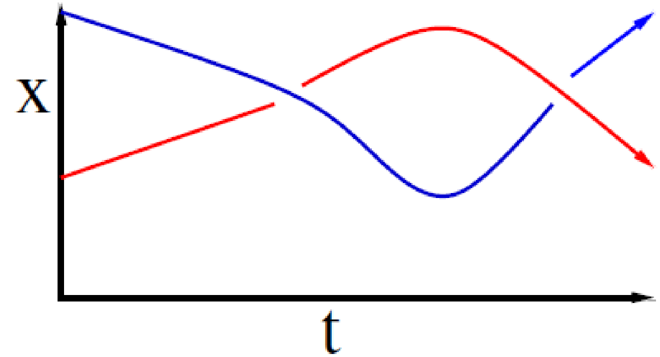
ML4Q

# Overview

- **Introduction:** Topological quantum computation and non-Abelian statistics
- **Chiral Majorana fermions**
- **Edge vortices = Ising anyons = flying Majorana zero modes (MZMs)**
- **AC Conductance of Majorana interferometer** can reveal the topological spin of edge vortices → evidence for **non-Abelian braiding of Ising anyons** from conductance measurements
- **Conclusions**

***A. Nava, R. Egger, F. Hassler, and D. Giuliano, arXiv:2403.03757***

# Anyon braiding




- Exchange of anyons ↔

Adiabatic interlacing of world lines in space-time („braiding“)

- Braiding Abelian anyons: only scalar phase factor in final state
- Braiding of non-Abelian anyons: final state related by unitary matrix to initial state
  - Topologically distinct braids (no continuous deformation without cutting world lines) ↔ different unitaries

    → building blocks for topological quantum computation
  - Braiding operations connect locally indistinguishable ground states → fault tolerance

# Majorana fermion primer

Consider set of Majorana fermions

- Self-adjoint operators with Clifford algebra

$$\gamma_i \gamma_j + \gamma_j \gamma_i = 2\delta_{ij}$$

  different Majoranas anticommute like fermions

- But: $\gamma_j^+ \gamma_j = \gamma_j^2 = 1$
  - annihilation of particle & antiparticle recovers previous state
  - Occupation number of single Majorana fermion ill-defined!
- We can count Majorana pairs: equivalent to conventional complex fermion $c = (\gamma_1 + i\gamma_2)/2$

  $\rightarrow$ single qubit $n = c^+ c = (i\gamma_1\gamma_2 + 1)/2 = 0{,}1$

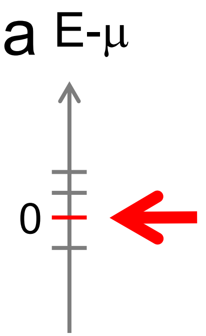
  $\gamma_1 = c + c^+, \ \gamma_2 = -i(c - c^+)$

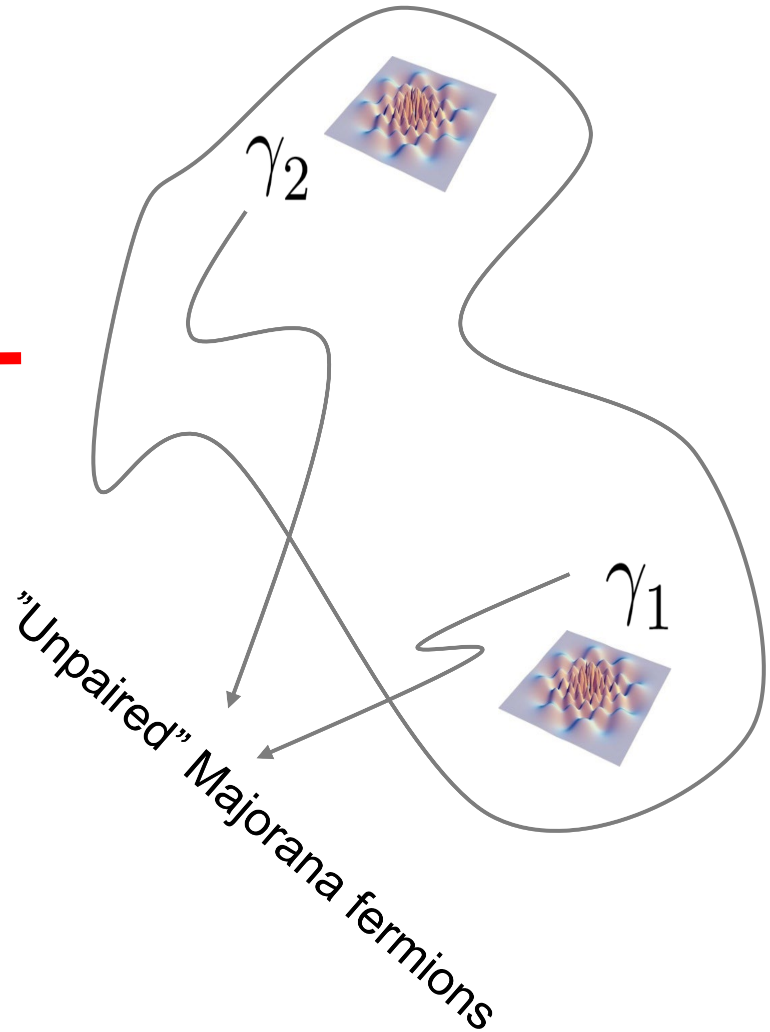
  equal-weight electron-hole superpositions

# Nonlocality and degeneracy

Spatially separate Majorana pair yields zero-energy fermion mode ↔ qubit

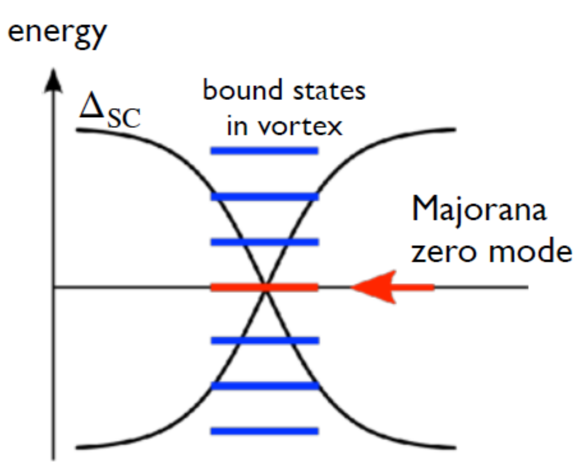
- Information stored non-locally & topologically protected

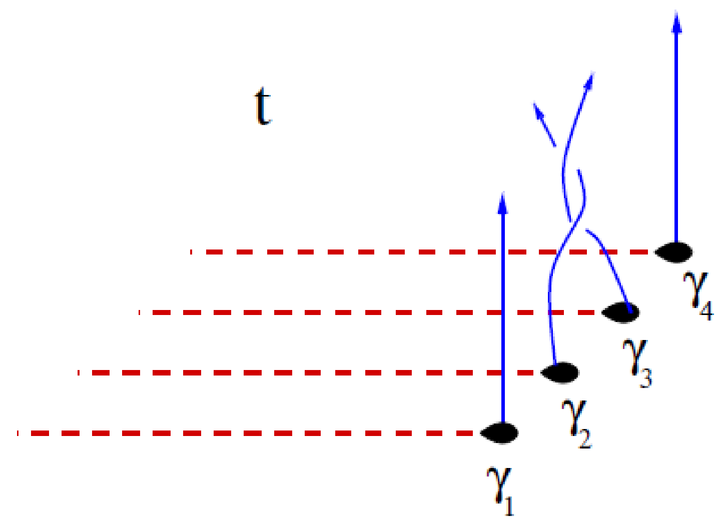


# Braiding Ising anyons

- Ising anyon = simplest type of non-Abelian particle = MZM locked to, e.g. vortex core in topological superconductor
- Mathematically, MZMs cause branch cuts → sign changes when cut is crossed during braiding operation
- For useful implementations, we need robust and reproducible hardware platform!



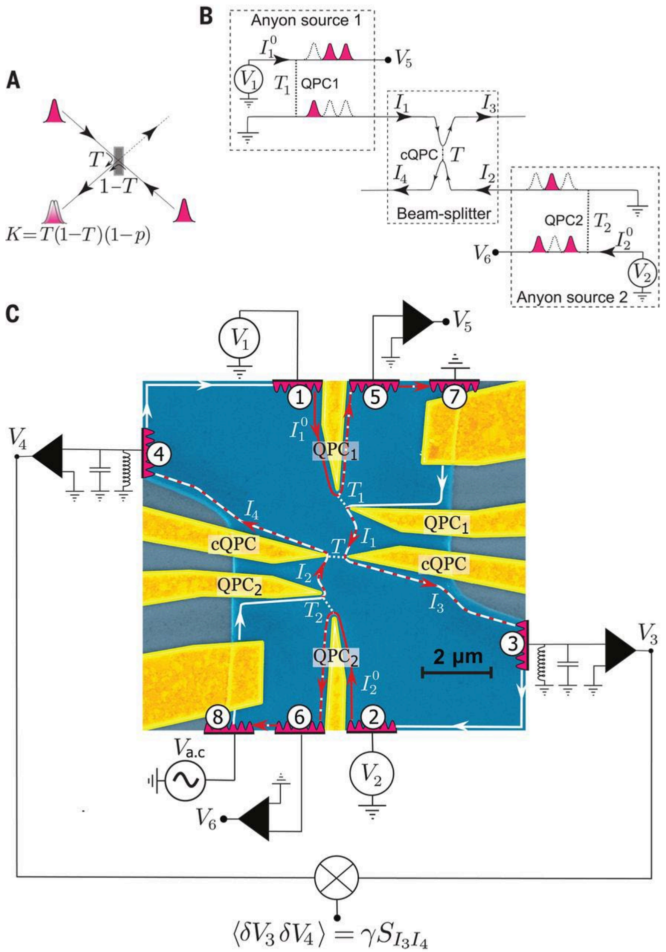



# Abelian fractional statistics

*Bartolomei, ... , Fève, Science 2020*
*Nakamura, ... , Manfra, Nature Phys. 2020*

- Tremendous progress in fractional quantum Hall samples (Abelian cases): **Shot noise & anyon collision** experiments have measured fractional **statistical exchange angles**
- Difficult experiments, took >20 years of development
- Abelian fractional exchange phases may be simpler to measure through conductance

*Schiller, Shapira, Stern, Oreg, PRL 2023*



# Non-Abelian braiding: How to observe it?

- So far only evidence for Majorana braiding from quantum simulations on transmon circuits *Stenger et al., Phys. Rev. Res. 2021, Harle et al., Nature Comm. 2023*

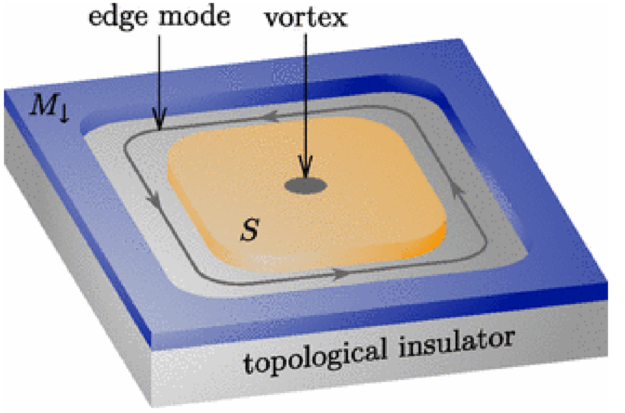
  → but not useful for practical topological quantum computation
- Problem with existing MZM platforms: disorder causes conventional subgap Andreev states → compete with MZM, hard to distinguish *Aghaee et al. (Microsoft Q), PRB 2023*
- Here: use analogy to (fractional) quantum Hall case → employ **edge vortex excitations of chiral Majorana fermion modes** as flying Ising anyons
- **Chirality protects intrinsically against disorder!**
- Which quantity to probe? Proposals: Shot noise in $\nu = \frac{5}{2}$ FQH interferometer *Bonderson et al., PRL 2006, Lee & Sim, Nat. Comm. 2022*

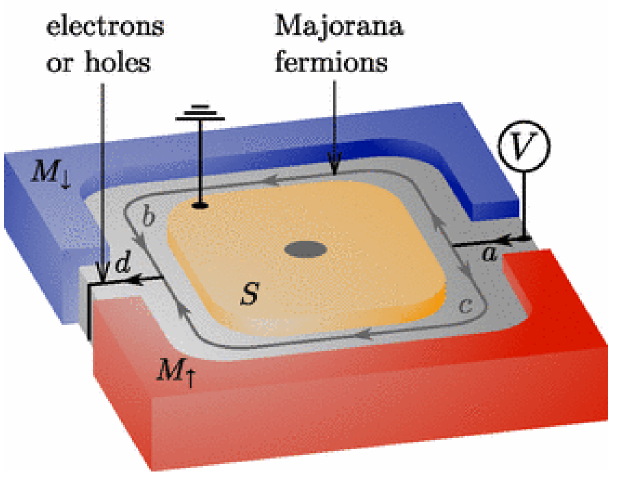
# Chiral Majorana interferometer

- Surface of 3D topological insulator = 2D gapless Dirac fermions (1 cone)
- Create gap by deposition of either superconductor (S) or magnet (M) → at interface: gapless 1D chiral Majorana mode (charge neutral)
- **Interferometer:** use unit-efficiency conversion of chiral Dirac fermions into pair of chiral Majorana fermions
- Copropagating Majorana modes!
- **Electrical DC conductance:**

$$G = (-1)^{n_v} \, e^2/h$$

probes number of bulk vortices




*Fu & Kane, PRL 2009*
*Akhmerov, Nilsson & Beenakker, PRL 2009*

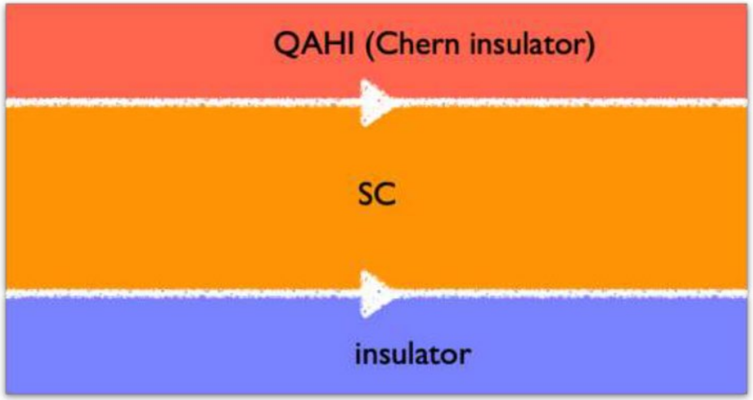
# Chiral Majorana edge modes

Edge Hamiltonian:

$$H_0 = -v \int dx\, \gamma(x) \partial_x \gamma(x)$$

Chiral Majorana fermions are Abelian (fermionic) particles with $\gamma(x) = \gamma^+(x)$

$$\{\gamma(x), \gamma(x')\} = \delta(x - x')$$




Non-Abelian Ising anyons:

**Chiral edge vortex $\sigma$** at $x = x_v$:

$$\sigma(x_v)\gamma(x)\sigma(x_v)^+ = \begin{cases} -\gamma(x), & x < x_v \\ \gamma(x), & x > x_v \end{cases}$$

# Edge vortices

- $\boldsymbol{\sigma} =$ domain wall for phase of chiral Majorana fermion mode
- Robust: Majoranas are real-valued
- Chirality: flows along with fermions at edge velocity
- Edge vortex binds a **MZM** (even though no „core“): **Ising anyon**
- Crucial difference to FQH interferometer: Majorana fermion modes (and edge vortices) are co-propagating → simpler schemes for accessing braiding statistics are possible
- Injection of deterministic (classical) edge vortices via fine-tuned flux pulses → time-domain charge measurements could give evidence for „guided braiding“ around bulk vortices

*Beenakker et al., PRL 2019; Adagideli et al., SciPost Phys. 2020*

- **Here:** study **quantum edge vortices**

# Quantum (dynamical) edge vortices

- No problems with in-gap states (no „normal“ core!)
- Insensitive to disorder (chirality!)
- Naturally movable non-Abelian particles, may be braided around static counterparts
- Topological spin $e^{2\pi i s_\sigma} = e^{\frac{i\pi}{8}}$ and conformal dimension $h_\sigma$ determine equal-time correlator:

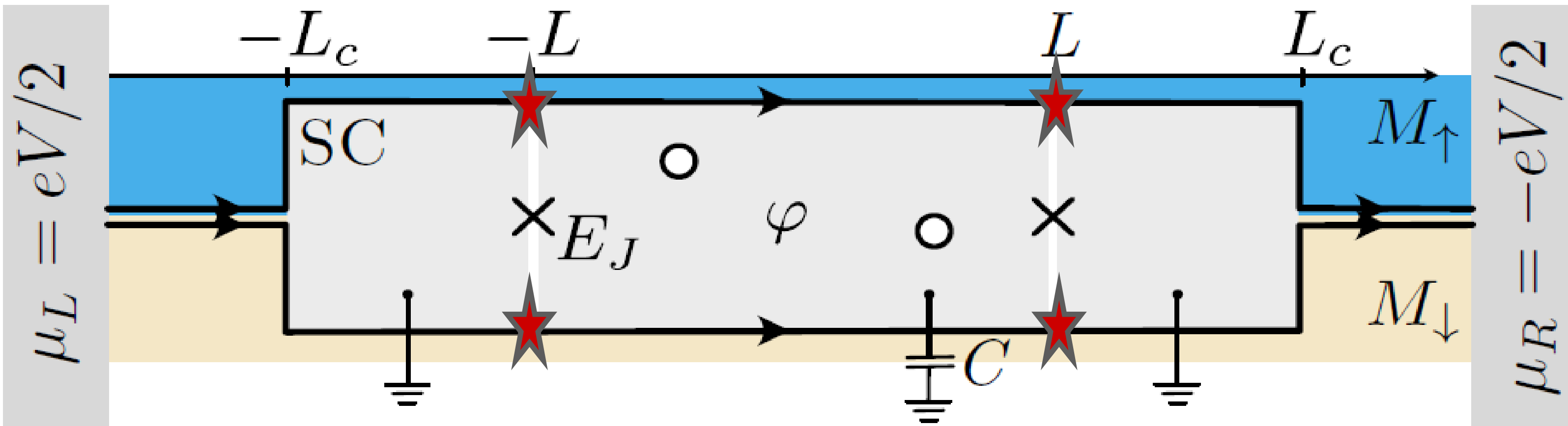
$$\langle \sigma(x)\sigma(0)\rangle \propto e^{2\pi i s_\sigma}|x|^{-2h_\sigma}, \qquad s_\sigma = h_\sigma = 1/16$$

Topological spin is related to non-Abelian braiding!

*Kitaev, Ann. Phys. 2006*

# Non-Abelian anyon interferometer

*Nava et al., arXiv 2024*




Add **central floating superconducting (SC)** island to Majorana interferometer & measure AC conductance between metallic leads

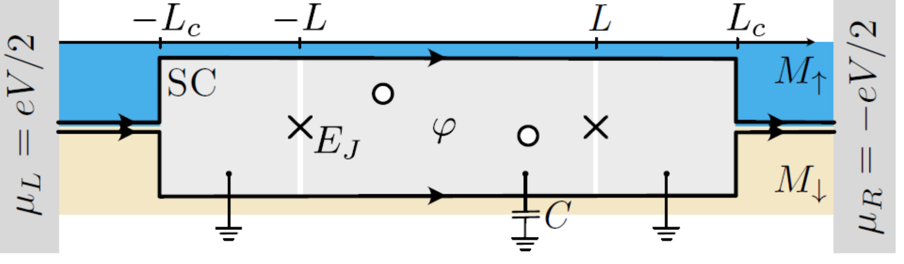
- two Josephson line junctions at $x = \pm L$ with Josephson energy $E_J$
- Central island: finite Coulomb charging energy $E_C \ll E_J \rightarrow$ fast phase slips $\varphi \rightarrow \varphi \pm 2\pi$ can generate four edge vortices ★

Phase slips occur at rate $\Gamma \approx \omega_p e^{-\sqrt{8E_J/E_C}}$ $\omega_p = \sqrt{8E_J E_C}$

plasma frequency

# Fine print: Model assumptions

- Plasma frequency $\omega_p \gg \Gamma, \Delta$ ($\Delta =$ induced SC pairing gap)
  - Phase slips are effectively time-local events
- Strip width $2W \gg \xi_0 = v/\Delta$ (SC coherence length)
  - Upper and lower Majorana modes don't hybridize except at junctions
- Neglect above-gap quasiparticles: low temperatures $k_B T < \Delta$ $\rightarrow$ transport through interferometer only via Majorana fermion modes (and $\sigma' s$) because of SC bulk gap
- Protected Dirac-Majorana conversion: include grounded SCs
- Equal path length on upper and lower arms (for now)



# Chiral bosonization: Key steps

- Combine both 1D chiral Majorana fermions to one 1D chiral Dirac fermion: $\Psi(x) = \frac{1}{\sqrt{2}}(\gamma_1(x) + i\gamma_2(x))$
- Bosonize Dirac fermion using chiral boson field $\phi(x)$:

$$\Psi(x) \propto \mathrm{e}^{-i\phi(x)}$$

$\rightarrow$ edge vortex operators are simple in bosonized language

- For edge vortex pair at $x = x_j$ (on top and bottom edge):

$$\sigma_t \sigma_b = S^- e^{\frac{i}{2}\phi(x_j)} + H.c.$$

  - Auxiliary spin ensures proper Ising anyon fusion rules consistent with CFT analysis *Fendley et al., PRB 2007*
  - Conserved $S_z = \pm\frac{1}{2}$ $\leftrightarrow$ total fermion parity conservation

# Euclidean functional integral

- To compute AC conductance in linear response, we proceed in imaginary time $0 \le \tau \le \beta = 1/T$
- Euclidean action (without voltage term): $S = S_0 + S_f + S_v$
- Free action of chiral boson field $\phi(x,\tau)$ is quadratic:

$$S_0 = \frac{1}{4\pi}\int d\tau \int dx \partial_x \phi [i\partial_\tau + v\partial_x]\phi$$

Majorana fermion tunneling action at Josephson junctions

$$S_f = \sum_j \frac{v\lambda_j}{2\pi} \int d\tau \partial_x \phi(x_j,\tau)$$ (include via unitary transformation)

Edge vortex creation/annihilation → nonlinear action:

$$S_v = \Gamma \int d\tau \cos\left[w_-(\tau) + 4\pi S_z s_\sigma + 2\pi n_g\right]$$

$$w_-(\tau) = \frac{1}{2}(\phi(L,\tau) - \phi(-L,\tau))$$

backgate charge offset parameter

# Quantum impurity problem

Integrate out all boson fields except for $w_\pm(\tau)$ via Lagrange multipliers (which are also integrated out)

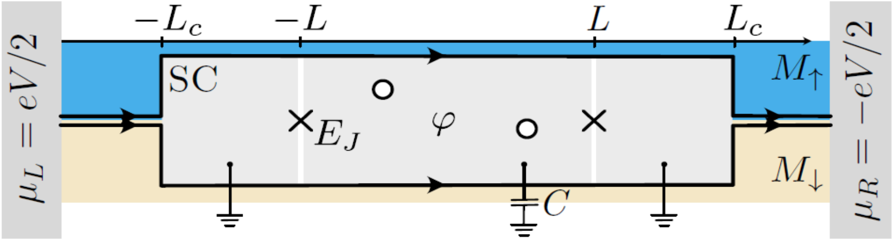
$$w_+(\tau) = \frac{1}{2}[\phi(L_c + W, \tau) + \phi(-L_c + W, \tau)]$$

$w_- \leftrightarrow$ charge fluctuations on central island

$\dot{w}_+ \leftrightarrow$ electric current through interferometer

**Linear response AC conductance** $G(\omega)$ from **Kubo formula** using $-i\Omega \to \omega + i0^+$ in equilibrium current-current correlator:

$$K(\Omega) = (-1)^{n_v} i\Omega \frac{e^2}{h} \langle \widetilde{w}_+(-i\Omega)\ \widetilde{w}_+(i\Omega)\rangle_S \qquad \widetilde{w}_+(i\Omega) = \int_0^\beta e^{-i\Omega\tau} w_+(\tau)\, d\tau$$



# AC conductance: Small $\Gamma$ regime

For $\Gamma \ll \max\left[T, \frac{v}{L}\right]$: nonlinearity $\propto \Gamma$ is RG-relevant

Perturbation theory in $\Gamma$ yields AC conductance (GHz regime)

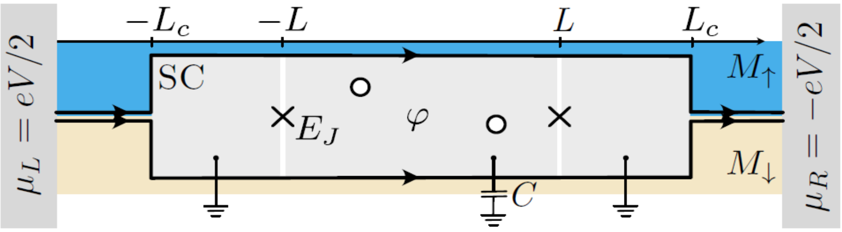
$$G(\omega) = G(0) + i(-1)^{n_v}\omega\left(L_{kin} - C_{eff}\right) + O(\omega^2)$$

**DC conductance**: $G(0) = (-1)^{n_v}\frac{e^2}{h}$ **unaffected** by fermion tunneling nor edge vortex tunneling

Kinetic inductance of Majoranas: $L_{kin} = \frac{e^2}{\pi v}(L_c + W)$

**Ising anyon statistics** appears in **effective capacitance**

(measurable through phase delay between current & voltage)

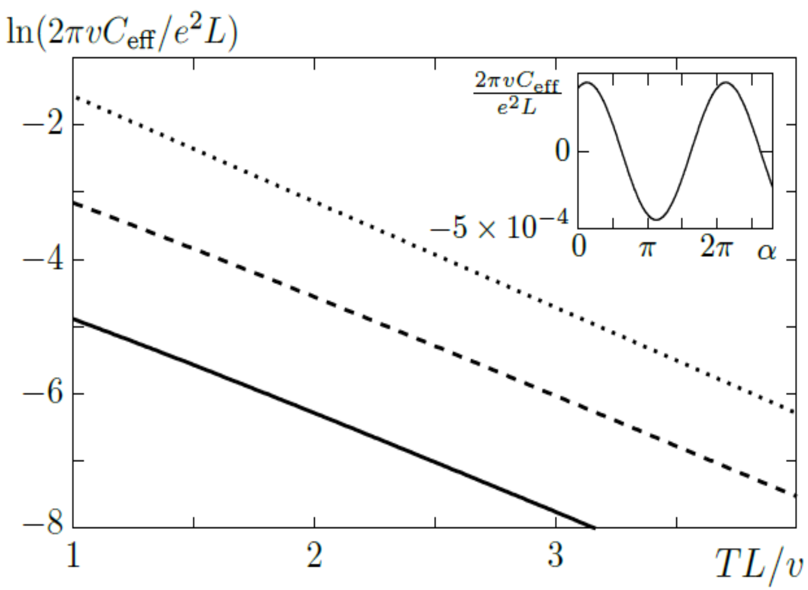


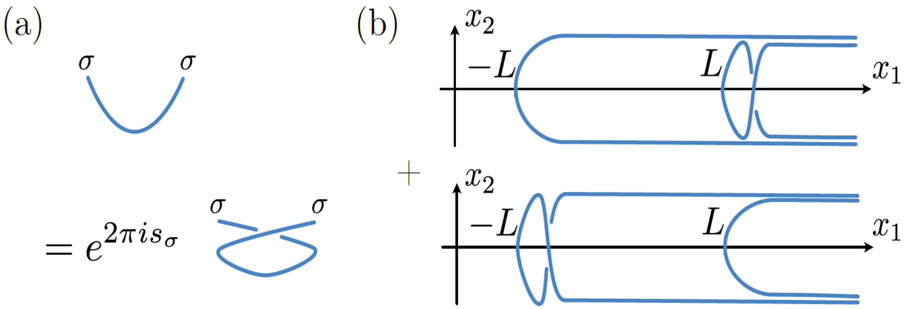
# Effective capacitance

Analytical result in perturbative regime $\Gamma \ll \max\left[T, \frac{v}{L}\right]$:

$$C_{eff} = \Gamma \frac{e^2 L^2}{2v^2} \cos(\boldsymbol{\alpha} - 4\pi S_z \boldsymbol{s_\sigma}) \left[\frac{\Delta}{T} \sinh \frac{2\pi T L}{v}\right]^{-4\boldsymbol{h_\sigma}}$$

$\boldsymbol{\alpha} = \frac{\pi(\lambda_1+\lambda_2)}{4} - 2\pi n_g$ **= tunable phase** (offset charge or finger gates at junctions)




*Kitaev, Ann. Phys. 2006*

**Oscillations in $\alpha$: anyon braiding**
(topological spin $s_\sigma$ appears!)
**Temperature dependence:** extract conformal dimension $h_\sigma$ from slope

# Conclusions

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- **Conclusions**

*A. Nava, R. Egger, F. Hassler, and D. Giuliano, arXiv:2403.03757*

**THANK YOU FOR YOUR ATTENTION!**