Majorana box qubit & Majorana surface code

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CRC183 ENTANGLED STATES OF MATTER

with Alex Altland, Karsten Flensberg, Aviad Landau, Stephan Plugge, Asbjørn Rasmussen & Eran Sela

Overview

- Majorana box qubit (MBQ)
  - Interacting Majorana-based topological qubit
  - Proof-of-principle MBQ experiments: initialization, readout, manipulation & entangling two MBQs
    Plugge, Rasmussen, Egger & Flensberg, NJP 2017, cf. Karzig et al. PRB 2017
- MBQ network as platform for Majorana surface code *Landau, Plugge, Sela, Altland, Albrecht & Egger, PRL 2016*
  - Topologically protected logical qubits
  - Basal logical qubit operations needed for universal quantum computation

Plugge, Landau, Sela, Altland, Flensberg & Egger, PRB 2016

Majorana bound states (MBSs)

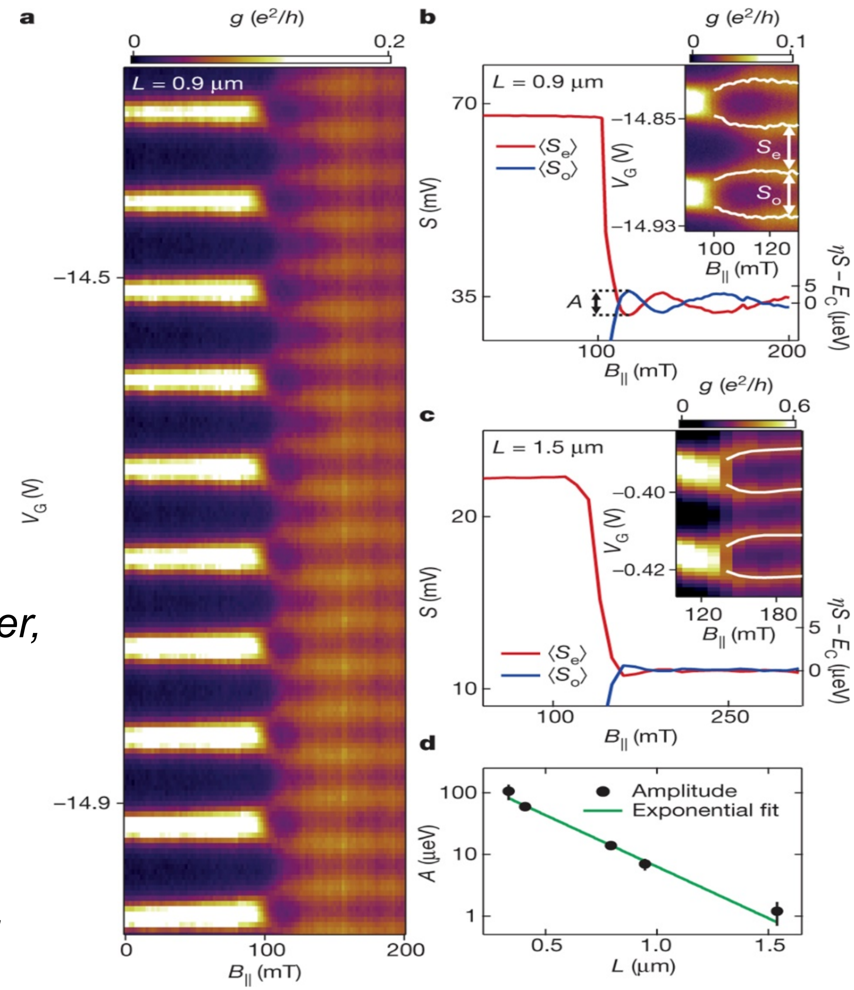
InAs (or InSb) **helical nanowires host MBSs** due to interplay of
- strong spin-orbit field
- magnetic Zeeman field
- proximity-induced pairing

Oreg, Refael & von Oppen, PRL 2010
Lutchyn, Sau & Das Sarma, PRL 2010

Coulomb blockade effects

Fu, PRL 2010; Hützen, Zazunov, Braunecker, Levy Yeyati & Egger, PRL 2012

MBS experiments*: Mourik et al., Science 2012, Rokhinson et al., Nat. Phys. 2012; Deng et al., Nano Lett. 2012; Das et al., Nat. Phys. 2012; Churchill et al., PRB 2013; Nadj-Perge et al., Science 2014; Deng et al. Science 2017 etc etc*



Albrecht et al., Nature 2016

Majorana basics

Consider set of MBSs at different locations in space

Self-adjoint operators $\gamma_j = \gamma_j^+$

Clifford algebra $\gamma_i \gamma_j + \gamma_j \gamma_i = 2\delta_{ij}$

- Different Majorana operators anticommute just like fermions
- But: $\gamma_j^+ \gamma_j = \gamma_j^2 = 1$
  - Occupation number of single MBS ill-defined
  - Pair of MBSs is equivalent to standard fermion mode

Majorana box

Beri & Cooper, PRL 2012
Altland & Egger, PRL 2013
Altland, Beri, Egger & Tsvelik, PRL 2014

Two helical nanowires proximitized by same mesoscopic floating superconducting island

on energy scales below proximity gap: neglect above-gap quasiparticles

$$H_{box} = E_C \left(\underset{\text{Electron number}}{N_c} - \underset{\text{gate parameter (close to integer)}}{n_g} \right)^2$$

- four MBSs (at zero energy for long wires)

  $\rightarrow$ two fermionic zero modes

- Condensate $\rightarrow$ bosonic zero mode

  $\rightarrow$ superconducting phase operator φ

Majorana box qubit (MBQ)

in Coulomb valley & for weak coupling to environment:

charge quantization → parity constraint

$$\gamma_1\gamma_2\gamma_3\gamma_4 = \pm 1$$

→ two-fold degenerate box ground state

(charge degree of freedom gapped out)

effective spin-1/2 = MBQ

non-locally encoded by topologically protected MBSs → long coherence times expected

Pauli operators: $\hat{x} = i\gamma_1\gamma_2, \quad \hat{y} = i\gamma_3\gamma_1, \quad \hat{z} = i\gamma_2\gamma_3$

spin fractionalization

Addressing the MBQ: Cotunneling

Tunneling Hamiltonian:
Connect MBSs to leads/dots

$$H_t = \sum_j \lambda_j \psi_j^+ e^{-i\varphi/2} \gamma_j + \text{h.c.}$$

- Respect charge conservation (floating device!)
  - Charge-neutral MBS dynamics vs change of box charge
  - MBS spin polarization encoded by tunnel matrix element
- Schrieffer-Wolff transformation

  $\rightarrow$ projection onto degenerate box ground state

Tunneling through box by **cotunneling**:

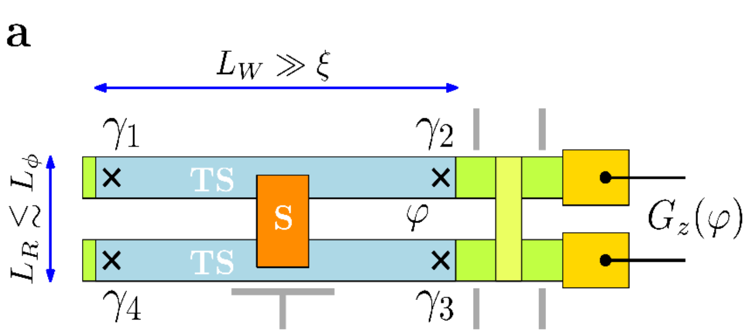
$$H_{2\leftrightarrow 3} = \psi_3^+ \left(t_0 + t_1 \hat{z}\right) \psi_2 + h.c.$$

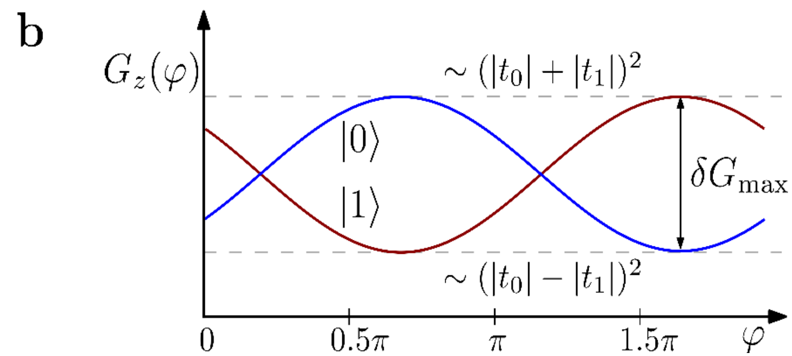
direct transition

$$t_1 \approx \frac{\lambda_3 \lambda_2^*}{E_C} \qquad \hat{z} = i\gamma_2\gamma_3$$

Readout & initialization

Tunnel conductance between normal leads attached to $\gamma_{2,3}$ & sharing reference arm → interference term sensitive to Pauli eigenvalue $z = \pm$ $\qquad G_z \sim \left| t_0 + t_1 z \right|^2$





- Two proximitized wires with conventional superconductor slab: Majorana box
- All wires parallel → homogeneous Zeeman field drives topological transition

→ **qubit initialization by projective readout**

Alternative readout via quantum dots

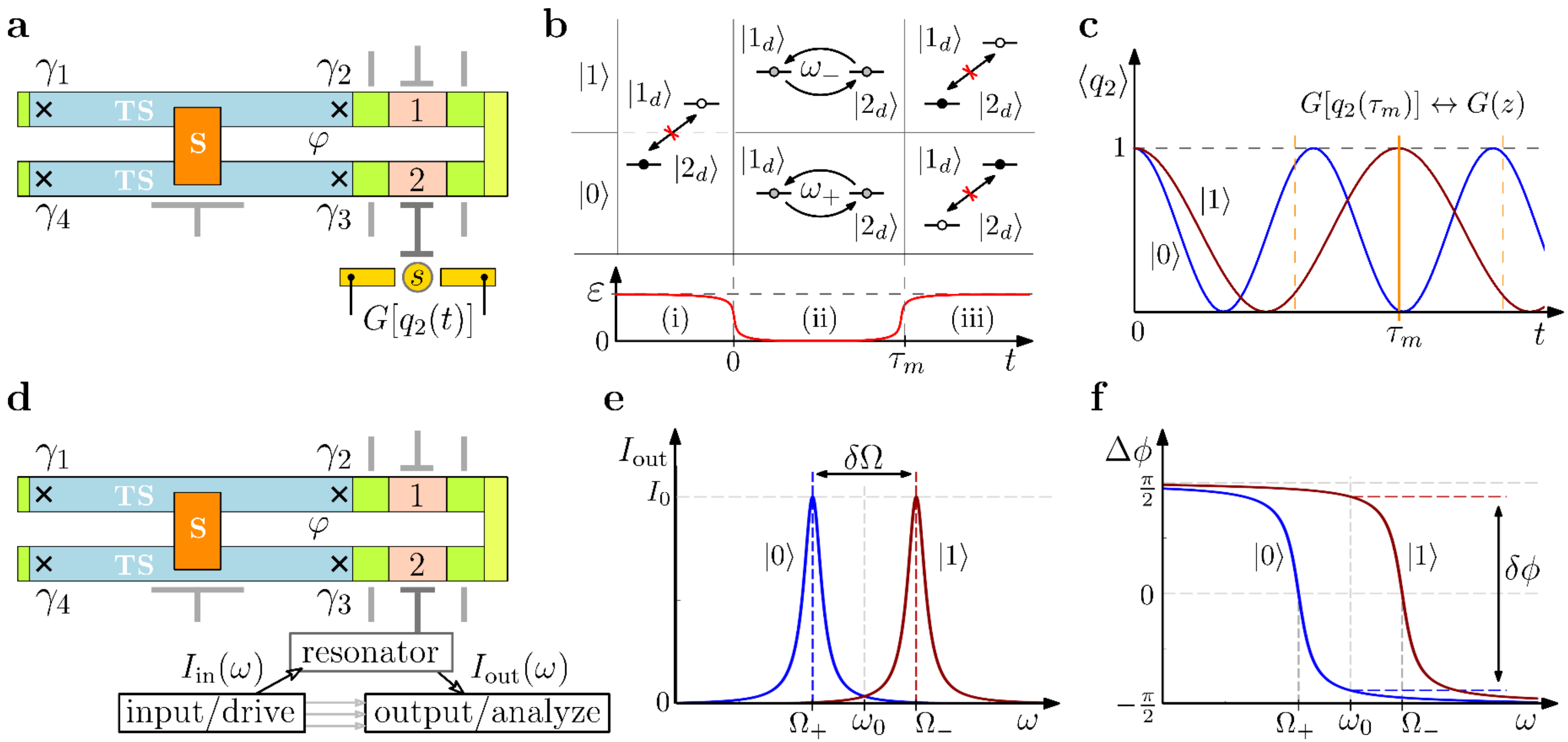
Replace leads by quantum dot pair in single-electron regime

Rabi oscillation frequency depends on MBQ state:

$$\omega_{z=\pm} = \sqrt{\varepsilon^2 + \left|t_0 + t_1 z\right|^2}$$

dot detuning

- Real-time charge detection on dot 2 (charge sensor)
- Alternatively: work in frequency domain using driven resonator



Qubit state manipulation

Confirmed single-electron pumping from dot 1→2 applies phase gate $\hat{P}(\theta) = e^{i\theta\hat{z}}$

$$\left|\psi\right\rangle \otimes \left|1_{dot}\right\rangle \rightarrow \hat{P}(\theta)\left|\psi\right\rangle \otimes \left|2_{dot}\right\rangle \qquad \theta = \tan^{-1}\left(\mathrm{Im}\frac{t_1}{t_0}\right)$$

robust operation, but fine tuning of tunneling phases necessary to avoid qubit dephasing

Turn off reference arm → Pauli operator: protected operation without any fine tuning

details of timing irrelevant **(adiabaticity not needed!)**, operation independent of tunnel couplings, etc.

Readout of all Pauli operators

need three quantum dots and one long reference arm

e.g. use another floating TS wire as long coherent link *Fu, PRL 2010*

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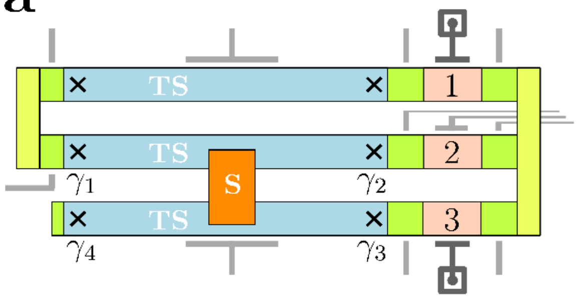

b readout / manipulation:

QDs 1 & 2 : $\hat{x} = i\gamma_1\gamma_2$

QDs 2 & 3 : $\hat{z} = i\gamma_2\gamma_3$

QDs 3 & 1 : $\hat{y} = i\gamma_3\gamma_1$

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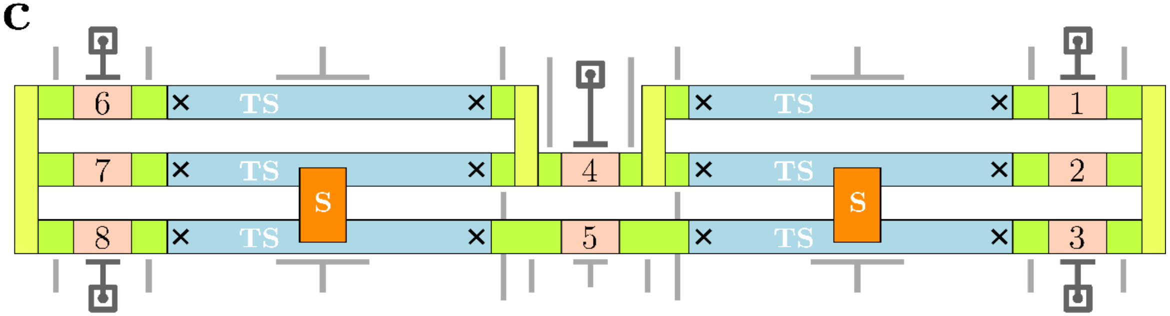

Qubit b Qubit a

QDs 4 & 5 : joint parity $\hat{z}_a\hat{z}_b$ $\longleftrightarrow$ $t_{4\rightarrow5} = t_a\hat{z}_a + t_b\hat{z}_b$

Entangling two qubits by measurement

1. Prepare eigenstates of $\hat{x}_{a,b}$
2. Measure joint parity: $\langle\hat{z}_a\hat{z}_b\rangle = \pm$ Entangled Bell state
3. Measure $\hat{x}_{a,b}, \hat{z}_{a,b}$ $\rightarrow$ Bell correlation tests

Four MBQ device: topologically protected Clifford gates

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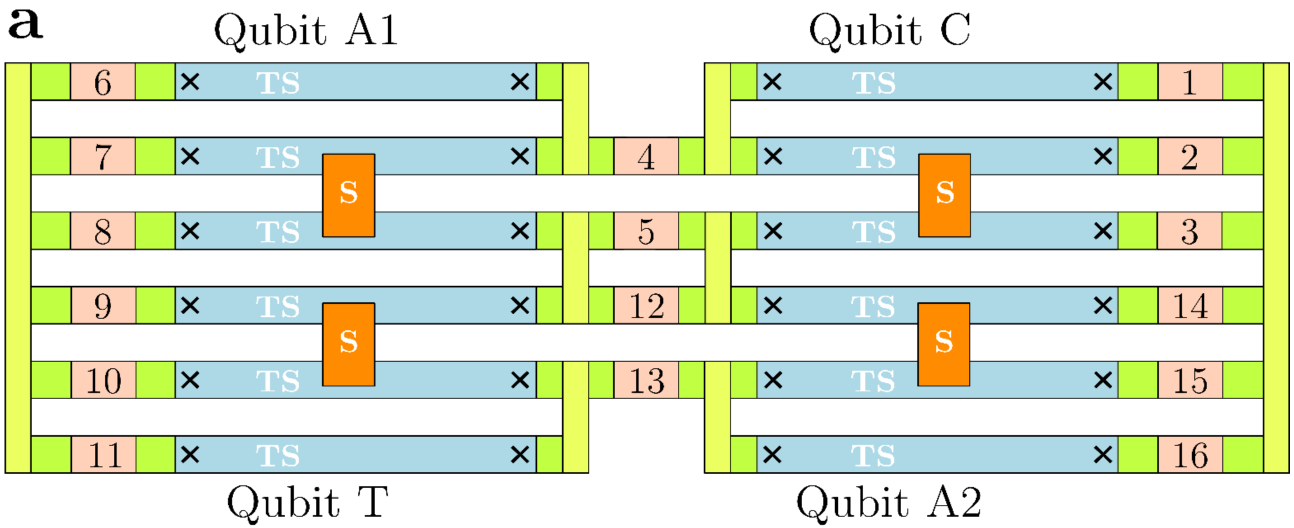
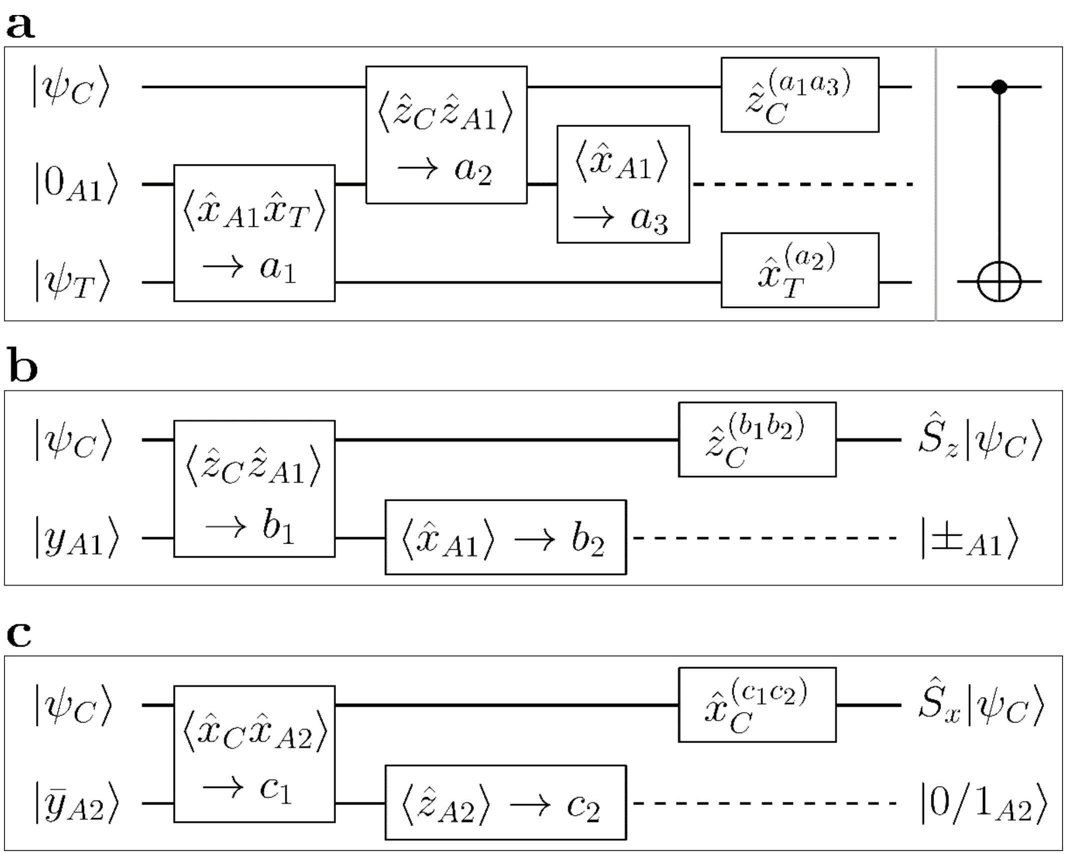


b

$1 \to 2$: $\hat{x}_C$	$6 \to 7$: $\hat{x}_{A1}$	$9 \to 10$: $\hat{z}_T$	$14 \to 15$: $\hat{z}_{A2}$
$2 \to 3$: $\hat{z}_C$	$7 \to 8$: $\hat{z}_{A1}$	$10 \to 11$: $\hat{x}_T$	$15 \to 16$: $\hat{x}_{A2}$
$1 \to 3$: $\hat{y}_C$	$6 \to 8$: $\hat{y}_{A1}$	$9 \to 11$: $\hat{y}_T$	$14 \to 16$: $\hat{y}_{A2}$
$4 \to 5$: $\hat{z}_C \hat{z}_{A1}$	$8 \to 9$: $\hat{x}_{A1} \hat{x}_T$	$12 \to 13$: $\hat{z}_T \hat{z}_{A2}$	$3 \to 14$: $\hat{x}_{A2} \hat{x}_C$

Measurement-based gate protocols



CNOT

cf. Preskill notes

S gate (z)

S gate (x)

Hadamard: $\hat{H} = \hat{S}_z \hat{S}_x \hat{S}_z$

→ protected Clifford quantum computer with 2 topological qubits

Topological qubits

So far: simple setups & protocols for proof-of-principle topological qubit experiments

Plugge et al., NJP 2017; Karzig et al., PRB 2017

- Qubit readout & initialization

  projective measurements using state-dependence of Rabi oscillation period between dot pairs
- Single-qubit operations

  confirmed single-electron pumping between dot pairs
- Protected Clifford operations for two qubit

Scaling to many MBQs? Fault tolerance?

→ **Surface code architecture**

Majorana surface code

- Interacting Majorana networks may realize Majorana surface code

  Terhal, Hassler & DiVincenzo, PRL 2012; Vijay, Hsieh & Fu, PRX 2015; Landau, Plugge, Sela, Altland, Albrecht & Egger, PRL 2016

- Surface code:
  - Scalable universal quantum computation scheme
  - Redundancy: Encode logical qubit by entangling many physical qubits in 2D arrays
  - Error tolerance orders of magnitude better than in alternative approaches

  review: Fowler, Mariantoni, Martinis & Clarke, PRA 2012

Majorana surface code principles

- **Repeat stabilizer readout in each cycle**

  Stabilizers = physical qubits = set of commuting operators built from elementary hardware qubits (here: four MBQs)

  → projection to highly entangled („stabilized") code state, cf. Kitaev toric code

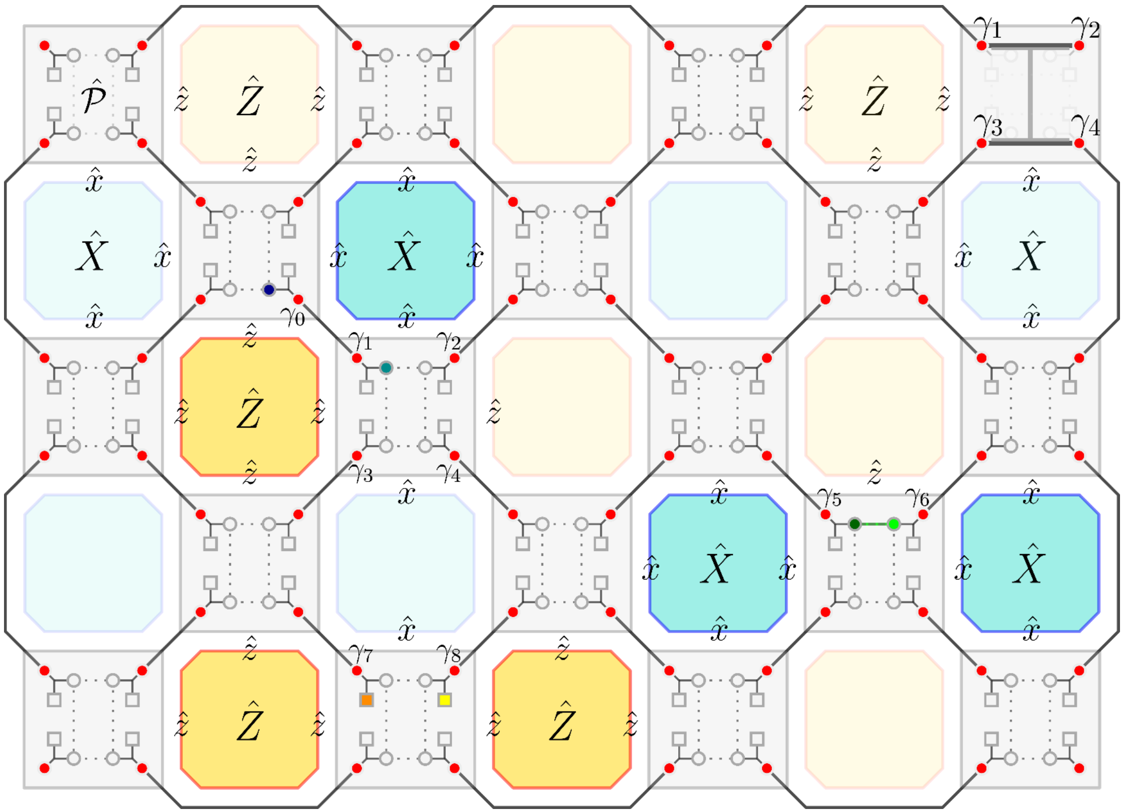
- Logical qubits: **stop measurements of specific stabilizers**

  Many physical qubits per logical qubit required...

  → simple & efficient basal quantum operations are needed

- Majorana formulation offers **unique simplifications**
  - No need for additional measurement qubits
  - Single-step stabilizer readout *Vijay, Hsieh & Fu, PRX 2015*
  - All-electric operations for semiconductor nanowire architecture *Landau et al., PRL 2016; Plugge et al., PRB 2016*
  - Does not require non-Abelian braiding of MBSs

Surface code architecture



MBQ = hardware qubit

$$\hat{x} = i\gamma_1\gamma_2$$

$$\hat{y} = i\gamma_3\gamma_1$$

$$\hat{z} = i\gamma_2\gamma_3$$

$$\hat{P} = \gamma_1\gamma_2\gamma_3\gamma_4 = \pm 1$$

Couple neighboring MBSs by gate-tunable **tunnel bridges** $t_{ll'}$

Two types of stabilizers = physical qubits

$$\hat{O}_A = \hat{X} = \hat{x}_1\hat{x}_2\hat{x}_3\hat{x}_4$$

$$\hat{O}_B = \hat{Z} = \hat{z}_1\hat{z}_2\hat{z}_3\hat{z}_4$$

Access elements: individual dot/lead coupled to each Majorana state with reference arms (coherent interference links) between nearby dots/leads

Stabilizers: Physical qubits

- Low-energy excitations of array correspond to minimal loop structures
  - Loop contains 8 Majorana operators = Product of 4 MBQ Pauli operators
- Hermitian plaquette operator for loop no. *n*

$$\hat{O}_n = \prod_{j=1}^{8} \gamma_j^{(n)}$$

- Set of mutually commuting operators
  - Stabilizer eigenvalues = ±1: simultaneously measurable set of physical qubits

Low-energy Hamiltonian

- Schrieffer-Wolff transformation yields

$$H_{code} = -\sum_n \mathrm{Re}(c_n)\,\hat{O}_n \qquad c_n = \frac{5}{16}\frac{\prod t_{l_n l_n'}}{E_C^3}$$

  - Amplitude contains product of 4 tunnel amplitudes in loop
  - Surface code works although prefactors are essentially random energies

- How to measure and manipulate stabilizers?

Essential ingredient for surface code quantum information processing, applied in every computation cycle

Interferometric stabilizer readout

Projective measurement

via Rabi oscillation period between dot pair attached to $\gamma_{0,1}$

Tunneling amplitude

$$t_{0\to1} = \xi + c_X^* \hat{X} + c_Z \hat{Z}$$

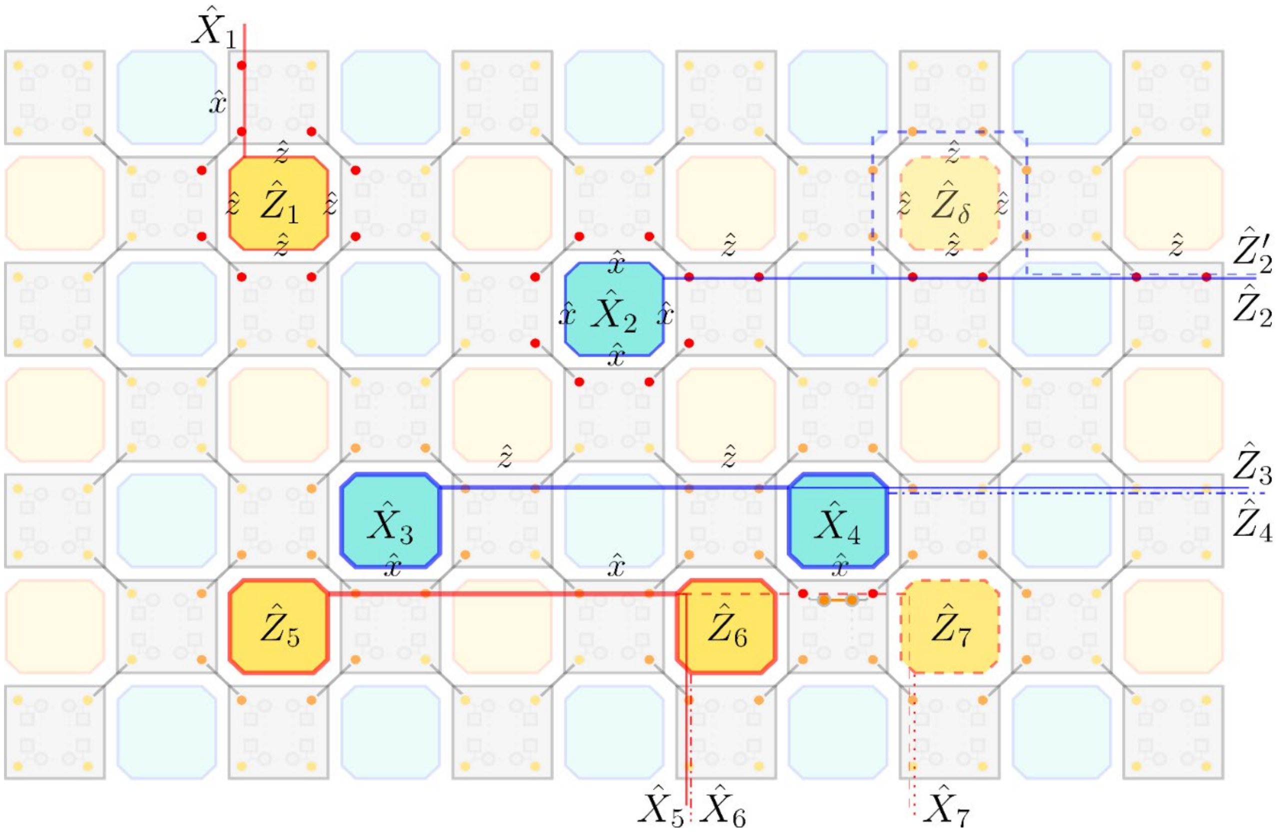
- two cotunneling paths around plaquettes
- direct amplitude ξ from 0→1 vanishes in Coulomb valley center (finite & tunable otherwise)

→ projection to state with eigenvalues $X, Z = \pm 1$

Logical qubits

„Stop measuring“ a given stabilizer:

1) Switch off MBS couplings λ_l to dots („software hole“)
2) Reduce intra-code $t_{ll'}$ couplings („hardware hole“) → degenerate qubit states



Logical qubits

Single-cut qubit $\left(\hat{Z}_1, \hat{X}_1\right)$

$\longrightarrow$ stop measuring stabilizer $\hat{Z}_1$

Anticommuting Pauli operator $\hat{X}_1$ is **string operator**

= product of MBQ Pauli-$\hat{x}$ operators

Double-cut qubit $\left(\hat{Z}_A, \hat{X}_A\right)$

$\longrightarrow$ stitch together two single-cut qubits $\hat{Z}_{5,6}$

redundant encoding: work in subspace $Z_5 Z_6 = +1$

Define $\hat{Z}_A = \hat{Z}_5$ & internal string operator $\hat{X}_A = \hat{X}_5 \hat{X}_6$

logical information decouples from boundaries!

... analogously for X-type logical qubits

Logical qubits

Hierarchy: Hardware qubits (MBQs) ⟹ Physical qubits (stabilizers) ⟹ Logical qubits

Using more holes for logical qubit: Code distance *d*

For elementary error probability below ≈1%, logical qubit errors exponentially (in *d*) suppressed

(fault-tolerance theorem) *Terhal, RMP 2015*

→ **topologically protected logical qubit**

Next: **initialize, manipulate, entangle & read out** logical qubits → minimal set of gates for universal quantum computation

Initialization of logical qubit $\left(\hat{Z}, \hat{X}\right)$

- Eigenstate of stabilizer operator $\hat{Z}$ follows after readout
- Eigenstate of Pauli-$\hat{X}$ operator by measuring string operator $\hat{X} = \gamma_1 \cdots \gamma_{2n} = \hat{x}_1 \cdots \hat{x}_n$

  $\rightarrow$ measure Rabi oscillations for dot pair attached to outermost Majoranas γ_1, γ_{2n} (with interference links)

Moving logical qubits: Same type

Single-cell move of logical information

Heisenberg: $\left(\hat{Z}_6, \hat{X}_6\right) \rightarrow \left(\hat{Z}_7, \hat{X}_7\right)$

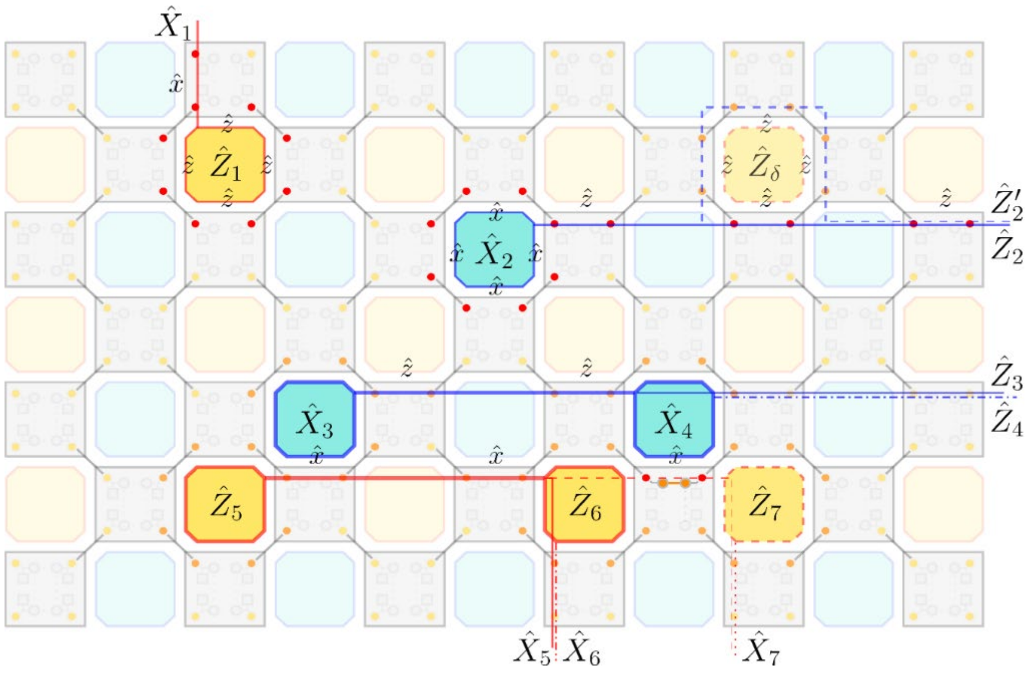
Schrödinger: $|\psi\rangle_6 \otimes |0\rangle_7 \rightarrow |0\rangle_6 \otimes |\psi\rangle_7$

for arbitrary $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$$\hat{Z}|0\rangle = |0\rangle$$
$$\hat{Z}|1\rangle = -|1\rangle$$
$$\hat{X}|+\rangle = |+\rangle$$
$$\hat{X}|-\rangle = -|-\rangle$$

Achieved by single-step measurement of string operator

$$\hat{X}_6 \hat{X}_7 = \hat{x}$$



Different types: Hadamard

Move between different-type logical qubits yields the **Hadamard gate**

$$\hat{H} = \frac{1}{\sqrt{2}}\left(\hat{X} + \hat{Z}\right)$$

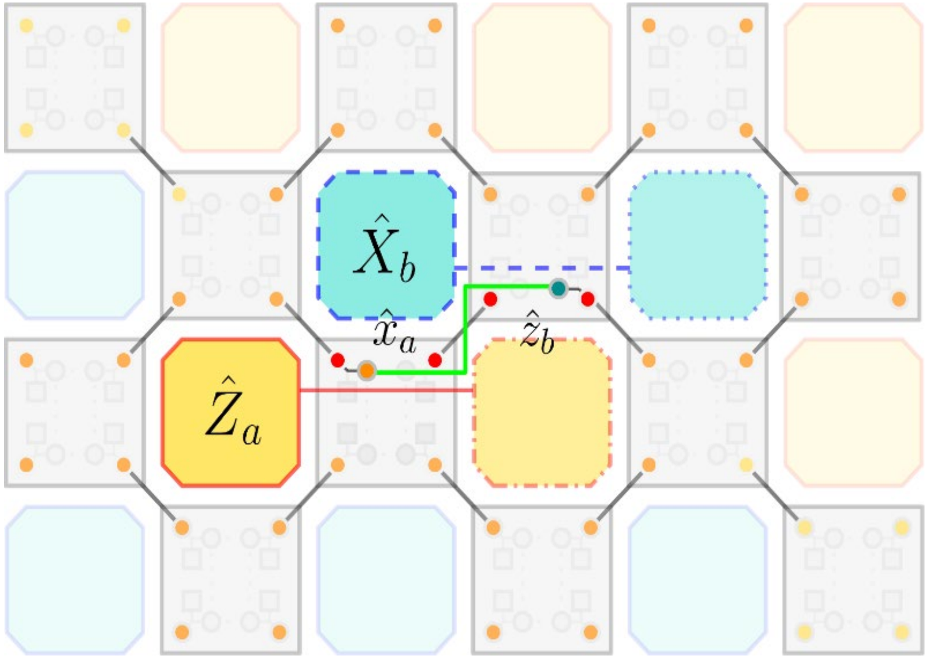
mapping $|0/1\rangle \leftrightarrow |+/-\rangle$

Implemented by measuring string operator

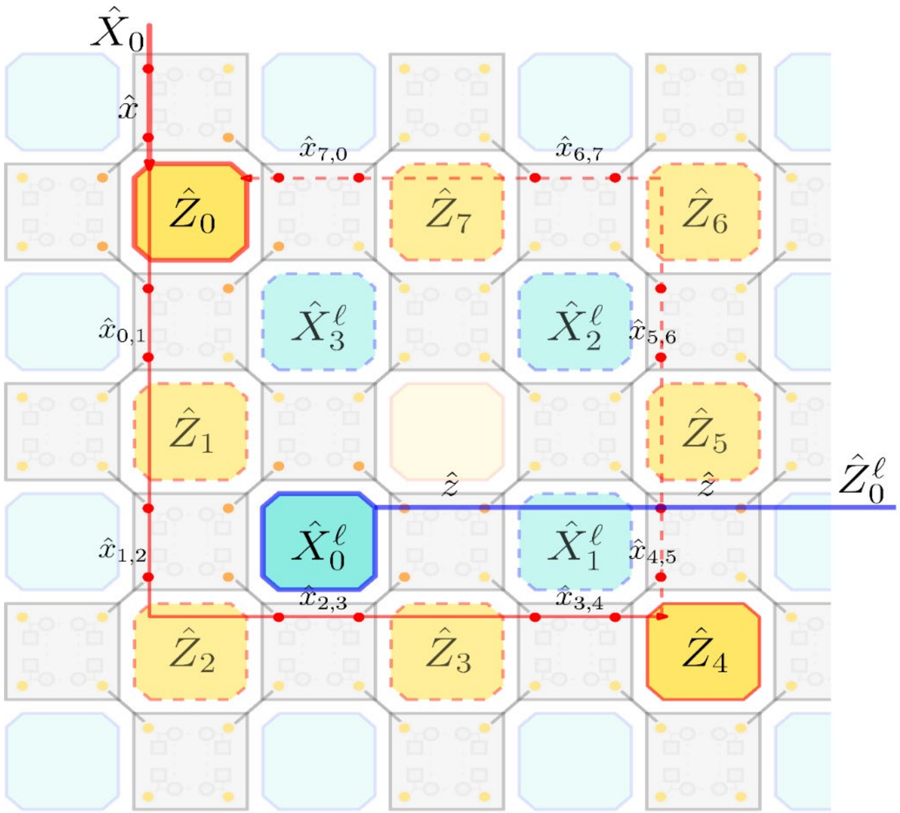
$$\hat{X}_a \hat{Z}_b = \hat{x}_a \hat{z}_b$$

$$\left(\hat{Z}_a, \hat{X}_a\right) \rightarrow \left(\hat{X}_b, \hat{Z}_b\right)$$

$$|\psi\rangle_a \otimes |+\rangle_b \rightarrow |0\rangle_a \otimes \left(\hat{H}|\psi\rangle\right)_b$$



Braiding two logical qubits: CNOT



Move Z-type qubit around X-type qubit by sequence of single-cell shifts: crossing of string line required $\rightarrow$ same transformation rules as **CNOT gate**

Fowler et al., PRA 2012
Vijay et al., PRX 2015

$$\left(\hat{X}_0\right)_{final} = \hat{X}_{loop}\left(\hat{X}_0\right)_{initial}$$

depends on state of enclosed qubit:

$$\hat{X}_{loop} = \hat{x}_{0,1}\cdots\hat{x}_{7,0} = \hat{X}_0^l\hat{X}_1^l\hat{X}_2^l\hat{X}_3^l \cong \hat{X}_0^l$$

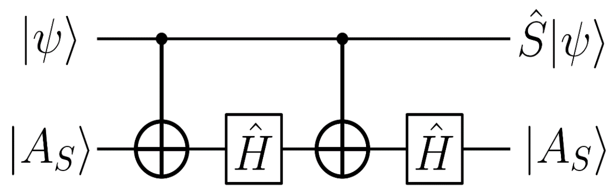
Universal quantum computation

Set of universal gates:

CNOT, Hadamard, and T gate

(plus S gate for convenience)

- CNOT: braiding X- and Z-type logical qubits
- Hadamard: moving X/Z- to Z/X-type logical qubits
- In addition, simple realization of S gate through two-qubit circuit



$$\hat{S}\left(\alpha|0\rangle+\beta|1\rangle\right)=\alpha|0\rangle+i\beta|1\rangle$$

Ancilla state $|A_S\rangle$ is Pauli- $\hat{Y}$ eigenstate

T gate implementation

So far only Clifford operations

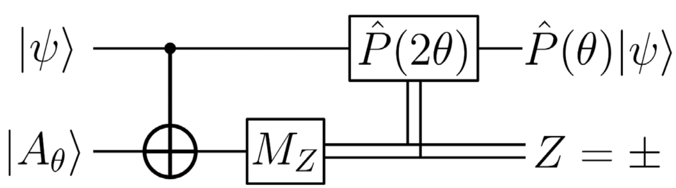
Missing phase gate $\hat{P}(\theta) = e^{i\theta\hat{Z}}$

e.g. T gate with $\hat{T} = e^{i\pi/8}\hat{P}\left(-\frac{\pi}{8}\right)$

$$\hat{T}\left(\alpha|0\rangle + \beta|1\rangle\right) = \alpha|0\rangle + e^{i\pi/4}\beta|1\rangle$$

$$\hat{S} = \hat{T}^2$$

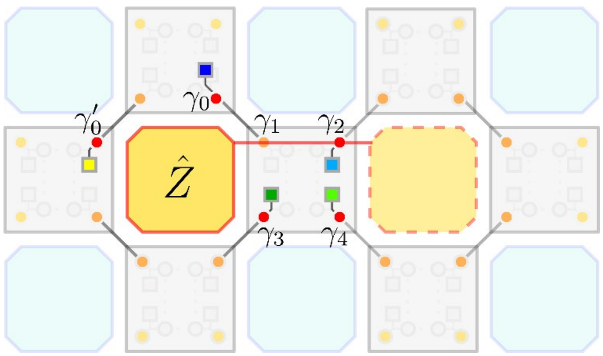
T gate is simplest phase gate sufficient for universality, follows from two-qubit circuit



Needed: ancilla state $|A_\theta\rangle = \hat{P}(\theta)|+\rangle = \frac{1}{\sqrt{2}}\left(e^{i\theta}|0\rangle + e^{-i\theta}|1\rangle\right)$

Multi-step pumping protocol

$\hat{X} = i\gamma_1\gamma_2$



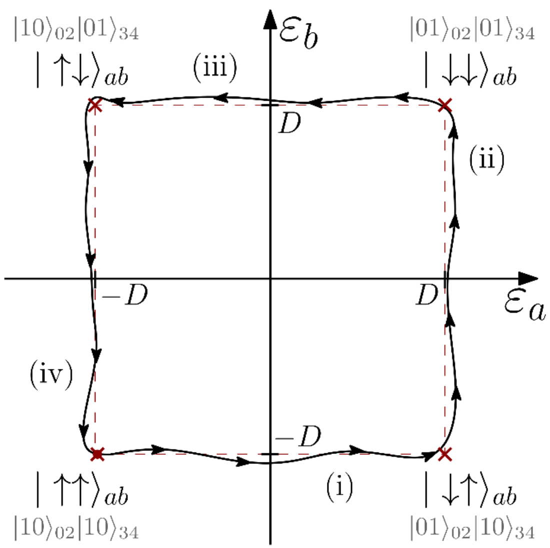
Ancilla preparation on qubit $(\hat{Z}, \hat{X})$

Start in string operator eigenstate $|+\rangle$

Apply **four-step pumping protocol** using dot pairs a=(0,2) and b=(3,4) by changing dot energies

$$\varepsilon_a / 2 = \varepsilon_0 = -\varepsilon_2$$

$$\varepsilon_b / 2 = \varepsilon_3 = -\varepsilon_4$$



Gate-steered qubit motion

Electron transfer through code by pumping via dot pair a=(0,2) [similarly for b=(3,4)]

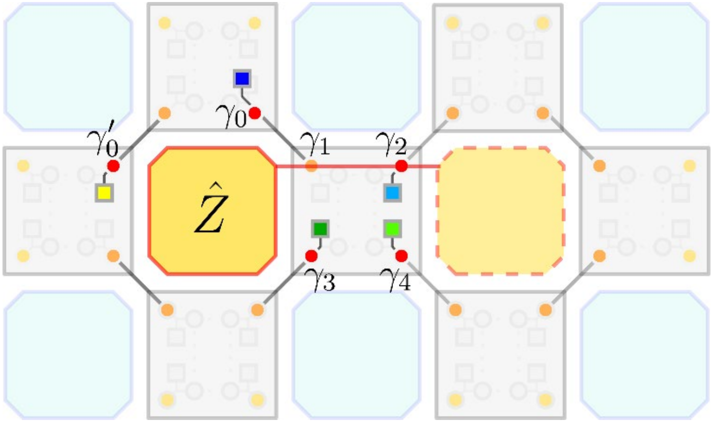
$$H_a = \psi_2^+ \left(i t_a^x \hat{X} + t_a^y \hat{Y} \right) \psi_0 + H.c.$$

Direct tunneling path

Excursion around stabilizer loop

Gate-steered motion of code qubit on Bloch sphere, running twice from north to south pole and back.

Dynamical phases cancel out exactly, only **geometric phase** θ remains



Final state is precisely $|A_\theta\rangle$

Geometric phase

- Protection: Protocol always yields phase gate, details of time dependence do not matter
- Static and tunable phase gate angle

$$\theta = \sum_{j=a,b} \tan^{-1}\left(\frac{2\left|t_j^x t_j^y\right|}{\left|t_j^x\right|^2 - \left|t_j^y\right|^2} \sin \chi_j \right) \approx \frac{E_C^2}{\bar{t}_{ll'}^2} \Delta n_g$$

Phase difference between t_j^x / t_j^y

Backgate detuning parameter off Coulomb valley center for box hosting γ_0

Conclusions

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  - Topologically protected logical qubits
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Plugge, Landau, Sela, Altland, Flensberg & Egger, PRB 2016

THANK YOU FOR YOUR ATTENTION